Nonlinearities in the relationship between debt and growth: (no) evidence from over two centuries*

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Abstract: I revisit the popular concern over a nonlinearity or threshold in the relationship between public debt and growth employing long time series data from up to 27 countries. My empirical approach recognises that standard time series arguments for long-run equilibrium relations between integrated variables (cointegration) break down in nonlinear specifications such as those predominantly applied in the existing debt-growth literature. Adopting the novel co-summability approach my analysis overcomes these difficulties to find no evidence for a systematic long-run relationship between debt and growth in the bivariate and economic theory-based multivariate specifications popular in this literature.

Keywords: public debt; economic growth; nonlinearity; summability and co-summability

JEL classification: H63, C22, E62, O40

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1 Introduction

"The latest research [by Reinhart and Rogoff] *suggests that once debt reaches more than about 90% of GDP the risks of a large negative impact on long term growth become highly significant."*

George Osborne, Mais Lecture, February 24, 2010

"The study [Reinhart and Rogoff (2010*b*)] *found conclusive empirical evidence that total debt exceeding 90 percent of the economy has a significant negative effect on economic growth."*

'The Path to Prosperity,' House Committee on the Budget, April 5, 2011

Despite the rhetoric adopted by a number of governments and opposition parties over recent years, determining a causal link from public debt to long-run growth as well as the potential nonlinearity of this relationship are widely regarded as unresolved empirical issues (International Monetary Fund, 2012; Panizza and Presbitero, 2013). As above quotes indicate the most influential research on the debt-growth nexus in recent years is unarguably the work by Reinhart and Rogoff (2010*b*) which has been adopted as justification for fiscal austerity measures by politicians on both sides of the Atlantic. Although recent revelations challenged the descriptive analysis carried out in their paper, Reinhart and Rogoff maintain that "the weight of the evidence to date — including this latest comment [by Herndon, Ash and Pollin (2014)] — seems entirely consistent with our original interpretation of the data" (*Wall Street Journal* 'Real Time Economics' blog, April 16, 2013), namely that "high debt/GDP levels (90 percent and above) are associated with notably lower growth outcomes" (Reinhart and Rogoff, 2010*b*, p.577; see also Rogoff, 2013). Perhaps aware of the tension between the causal interpretation typically read into this type of statement and the descriptive nature of their analysis, some of their earlier work (Reinhart, Reinhart and Rogoff, 2012) already pointed to a set of empirical studies which are argued to address both concerns regarding causality and identification of a nonlinearity in the *long-run* debt-growth relationship (e.g. Kumar and Woo, 2010; Balassone, Francese and Pace, 2011; Cecchetti, Mohanty and Zampolli, 2011; Checherita-Westphal and Rother, 2012)[1] in support of their findings.

[1]A further empirical study by Baum, Checherita-Westphal and Rother (2013) is cited in Reinhart, Reinhart and Rogoff (2012) but argues to focus on the *short-run* relationship. They note that their sample selection is driven by the finding that data for 1990–2007 appears stationary, whereas the longer 1980–2007 data appears nonstationary.

This paper investigates the debt-growth nexus from a new angle and with a somewhat more modest aim, focusing on the persistence of the long time series data used in the original Reinhart and Rogoff (2010*b*) study. I adopt annual data for over two centuries (1800–2010) to investigate whether linear or various nonlinear specifications of the debt-growth nexus constitute 'long-run equilibrium relations' in four OECD countries: the United States, Great Britain, Japan and Sweden;[2] additional work presented in an Online Appendix extends the analysis to 27 advanced and developing economies. The analysis employs the most popular specifications in this empirical literature – polynomial functions and piecewise linear (threshold) specifications – to model the hypothesised nonlinearity. The basic premise of my analysis is that if variable series are integrated (nonstationary), then the popular implementations of nonlinearity in the debt-growth literature (squared debt terms or debt terms interacted with threshold dummies) are invalid, since these transformations of the variable are not defined within the (co-)integration framework. Therefore any empirical results building on these polynomial or threshold specifications may be spurious.

My empirical strategy addresses this problem by adopting novel methods for summability and co-summability testing (Berenguer-Rico and Gonzalo, 2014*a*,*b*). These concepts provide a framework encompassing integration and cointegration which however extends to non-linear relationships. The analysis in this study is thus (narrowly) focused on the question of potential nonlinearities in the long-run debt-growth relationship, bypassing any concerns over the direction of causation *which does not impact the statistical validity of the results*. If there is no evidence for (nonlinear) long-run relations, then standard empirical specifications in the literature adopting thresholds or polynomial functions are misspecified and the causal interpretation assigned in these studies is questionable: the presence of a long-run equilibrium is a pre-requisite for the existence of any long-run causal relationship in the data. Results have important policy implications given that the most vocal supporters of fiscal austerity have pointed to the above-cited studies as providing empirically sound evidence for (this type of) nonlinearities in the debt-growth relationship.

The primary empirical focus is to investigate data for debt and GDP in Great Britain, Japan, Sweden and the United States over the 1800-2010 time horizon. In additional analysis I investigate

[2]The former two economies are presently at the centre of a policy debate relating sustainable growth to fiscal austerity (e.g. US Senate Budget Committee, June 4, 2013), Japan is at times taken as an example for sustained growth at comparatively high levels of debt, while Sweden (alongside the US and Britain) represents the country with the longest time series in my matched dataset.

sub-periods of 60 years using rolling window analysis to allow for structural breaks in the debt-growth relationship and also to reduce the impact of global shocks such as World War II on the presence or absence of a long-run relationship. A host of further robustness checks are confined to an Online Appendix.

My core analysis finds no evidence for any long-run relationship between debt and growth in the linear or nonlinear specifications for the four countries investigated. Subsample analysis does not fundamentally challenge this finding although it provides an indication that there may have been long-run relationships between debt and growth at different points in time, although not in the post-WWII period typically studied in the existing literature. The general patterns revealed by the subsample analysis supports the notion that the debt-growth relationship differs across countries and "with economic circumstances" (Larry Summers, Witness Statement to the US Senate Budget Committee, June 4, 2013). Additional empirical analysis goes to great lengths to determine whether the choice of countries, time periods, and/or atheoretical specifications drive this finding but arrives at a fairly consistent picture across all different modes of investigation. These findings help challenge the apparent consensus in parts of the empirical literature of both the *existence* and the *common* nature of a debt threshold across countries.

The remainder of the paper is organised as follows: the next section discusses the existing literature on debt and growth, with Section 3 providing the theoretical background for my econometric approach. Section 4 introduces the data and describes the debt-growth nexus in each of the four OECD countries which are at the core of my analysis. Results for these and a larger set of countries are presented and discussed in Section 5, before Section 6 concludes.

2 Existing Literature

The existing empirical literature on the debt-growth nexus builds on somewhat ambiguous theoretical foundations (see Panizza and Presbitero, 2013, for a recent survey). Some theoretical models argue that higher stocks of public debt may create increased uncertainty or even fear of future financial repression among investors and thus lead to a negative long-run relationship (Elmendorf and Mankiw, 1999; Teles and Cesar Mussolini, 2014) between debt and growth. Other work maintains that this negative relationship disappears once sticky wages and unemployment are taken into account in the modelling process (Greiner, 2011). The nonlinearity or debt threshold hypothesised and investigated in most empirical work can be motivated for developing countries by pointing to the issue of debt overhang (Krugman, 1988; Sachs, 1989), although it may be difficult to extend this argument to advanced economies such as those investigated in this paper. Nonlinearities may further arise if there is a tipping point of fiscal sustainability as is developed in Ghosh et al. (2013), however I am not aware of any theoretical models incorporating such debt tipping points into a framework for economic growth over the long-run.

As was suggested above, the work by Reinhart and Rogoff (Reinhart and Rogoff, 2009, 2010*a*,*b*, 2011) is largely descriptive in nature, although this should not distract from the significant contribution these authors have made to the literature in the construction of long data series for empirical analysis. Regression analysis of the debt-growth nexus conducted using panel data typically shares the unease about misspecification and endogeneity with the wider cross-country growth literature (see Durlauf, Johnson and Temple, 2005; Eberhardt and Teal, 2011, for a discussion of the latter). Empirical specifications in this literature are across the board partial adjustment models in the mould of Barro (1991) and Mankiw, Romer and Weil (1992) – regressing growth on a lagged level of per capita GDP and a measure for debt stock as well as typically a large number of control variables – in a pooled model specification, thus assuming away the possibility of parameter heterogeneity across countries.[3] The standard practice in the cross-country literature to average data over three- or five-year intervals in the panel is also adopted in all but the most recent papers (Checherita-Westphal and Rother, 2012; Baum, Checherita-Westphal and Rother, 2013; Panizza and Presbitero, 2014). Samples differ significantly across existing studies, with the work by Kumar and Woo (2010), Cecchetti, Mohanty and Zampolli (2011), Checherita-Westphal and Rother (2012), Baum, Checherita-Westphal and Rother (2013) and Panizza and Presbitero (2014) primarily focused on OECD and other high-income economies and thus most relevant to this study. Among these OECD country studies the only one to adopt a polynomial specification is the paper by Checherita-Westphal and Rother (2012), although this practice is popular in the study of developing economies (e.g. Cordella, Ricci and Ruiz-Arranz, 2010; Calderon and Fuentes, 2013;

[3]Notable exceptions include studies by Henderson and Parmeter (2013) and Kourtellos, Stengos and Tan (2013) which emphasise the heterogeneity of the debt-growth nexus across countries and adopt nonparametric methods to identify a threshold in the cross-section dimension.

Presbitero, 2012). With the exception of Cecchetti, Mohanty and Zampolli (2011), who apply the within (fixed effects) estimator and thus cannot address concerns over reverse causality, all of the above empirical studies implement their panel analysis adopting the Blundell and Bond (1998) System GMM estimator originally developed for firm-level panel data analysis.[4]

Despite different sample periods, country coverage, control variables, modelling of the nonlinearity and choice of moment conditions for identification, these studies come to remarkably similar conclusions, namely that beyond a threshold at around 90% debt-to-GDP the relationship between debt and growth is negative significant. However, as demonstrated by Panizza and Presbitero (2013), these findings are either not robust to small changes in the sample, suggesting the results are driven by outliers, or fail to formally test the coefficients on the pairwise linear terms, which on closer inspection typically cannot support the notion of a statistically significant change in the debt coefficient above the threshold.

All of the above studies are focused on pooled panel data modelling, implicitly assuming that the long-run equilibrium relationship between debt and growth is the same for all countries in the sample. Existing research has found very different results when moving away from full sample analysis in homogeneous parameter regression models and toward sub-sample analysis along geographic, institutional or income lines (International Monetary Fund, 2012; Kourtellos, Stengos and Tan, 2013; Eberhardt and Presbitero, 2015). There are a number of reasons to assume the equilibrium relationship between debt and growth could differ across countries. Vulnerability to public debt depends not only on debt levels, but also on debt composition (Inter-American Development Bank, 2006). Unfortunately, existing data for the analysis of debt and development often represent a mix of information relating to general and central government debt, debt in different currency denominations and with different terms attached (be they explicit or implicit). All of this implies that comparability of the debt data across countries may be compromised (Panizza and Presbitero, 2013). In addition, even assuming that debt stocks are comparable across countries and over time, the possible effect of public debt on GDP may depend on the reason *why* debt has been accumulated and on whether it has been consumed or invested (and in the latter case in which

[4] A thorough critique of this implementation in the macro panel context is beyond the scope of this paper. Eberhardt and Teal (2011) highlight the problems arising, Bun and Sarafidis (2013) provide an analysis of the impact of nonstationary initial conditions on this set of estimators while Pesaran and Smith (1995) discuss the bias arising from heterogeneity misspecification.

economic activities). Furthermore, different stocks of debt may impinge differently on economic growth: debt can clearly hinder GDP growth when it becomes unsustainable, affecting interest rates and triggering a financial crisis, thus affecting the level of GDP. However, the capacity to tolerate high debts depends on a number of country-specific characteristics, related to past crises and the macro and institutional framework (Reinhart, Rogoff and Savastano, 2003; Kraay and Nehru, 2006; Manasse and Roubini, 2009). For these reasons the focus of analysis in this paper is on country-by-country investigation of the long-run relationship between debt and growth.

A recent study which empirically investigates the debt-growth nexus with a time series econometric approach is the paper by Balassone, Francese and Pace (2011) on Italy (1861-2009). Adopting unit root and cointegration testing prior to estimation they establish a long-run relationship between per capita GDP, per capita capital stock and debt-to-GDP ratio (all in logarithms). They then go on to estimate (among other models) a piecewise linear specification for the debt-to-GDP ratio where values beyond a threshold of 100% are found to create a significantly stronger negative effect on growth – it is precisely this form of interaction between a threshold dummy and the debt-to-GDP ratio which is not defined under (linear) cointegration and which necessitates the present analysis.[5] It should also be noted that cointegration does not imply causation from debt to growth.

3 Nonlinear Relations between Integrated Processes

3.1 Methodology

In this section I highlight the difficulties arising for conventional time series analysis when assuming a non-linear model in the presence of integrated variables and discuss a novel approach to tackle these issues.

Suppose a time series relationship $y_t = f(x_t, \theta) + u_t$ for a nonstationary regressor $x_t \sim I(1)$, stationary u_t and some non-linear function $f(\cdot)$. Assuming for illustration $f(x_t) = \theta_1 x_t + \theta_2 x_t^2$,

[5] Adopting the threshold specification I find that in my data series for Italy none of the various thresholds adopted pass the co-summability test (100% threshold CI low 0.313, $\hat{\delta}_{\hat{e}_t}$=1.134, CI up 1.954; 90% CI low 0.486, $\hat{\delta}_{\hat{e}_t} = 1.052$, CI up 1.619; 70% CI low 0.910, $\hat{\delta}_{\hat{e}_t} = 1.695$, CI up 2.480; 50% CI low 0.811, $\hat{\delta}_{\hat{e}_t} = 1.471$, CI up 2.130) – see results section for notation.

let $x_t = x_{t-1} + x_0 + \varepsilon_t$ and $\varepsilon_t \sim i.i.d.(0, \sigma^2_\varepsilon)$, then we know that

$$\mathbb{V}[x_t - x_{t-1}] \;=\; \sigma^2_\varepsilon \quad \Rightarrow x_t \sim I(1) \tag{1}$$

In words, it can be shown that the Engle and Granger (1987, henceforth EG) characterisation of a stationary process holds for Δx_t – finite variance is one of five EG characteristics. Now investigate the same property for Δx_t^2:

$$\mathbb{V}[x_t^2 - x_{t-1}^2] \;=\; \mathbb{E}[\varepsilon_t^4] + 4(t-1)\sigma^4_\varepsilon - \sigma^4_\varepsilon \quad \Rightarrow x_t^2 \sim I(?)$$

Here the finite variance characteristic is clearly violated, given that the variance is a function of time. Since this problem cannot be solved by further differencing it is not possible to determine the order of integration of x_t^2. This in turn creates fundamental problems if the empirical analysis of $y_t = \theta_1 x_t + \theta_2 x_t^2 + u_t$ is to be based on arguments of cointegration. The difficulty arises from the requirement of the EG characterisation to investigate the *differences* of a process, with the intrinsic linearity of the difference operator creating obvious problems for nonlinear processes.

The following briefly introduces a novel set of methods for nonlinear processes which closely resemble the standard toolkit in linear time series analysis (tests for unit root behaviour and cointegration). The motivation for these new methods is to create "a summary measure of the stochastic properties – such as persistence – of the time series without relying on linear structures" (Berenguer-Rico and Gonzalo, 2014*b*, 3). The implementation of these tests is straightforward, involving OLS regressions of transformed variable series, where transformations avoid the first differencing so central to Dickey-Fuller-type unit root analysis and instead build on running sums. Like in the case of unit root analysis the distributions of these test statistics are non-standard, but estimates for sub-samples can be used to create confidence intervals for inference.

Berenguer-Rico and Gonzalo (2014*b*) build on earlier work by Gonzalo and Pitarakis (2006) to develop a non-linear alternative to linear integration, based on the 'order of summability.'[6] The empirical procedure to determine the order of summability analyses the rate of convergence of a rescaled sum Y_k^* of the variable of interest y_t. Using least squares we can estimate for k =

[6]For a formal definition of summability see Definition 2 in Berenguer-Rico and Gonzalo (2014*b*).

$1, \ldots, T$

$$Y_k^* = \beta^* \log k + U_k^* \quad (2)$$

where $Y_k^* = Y_k - Y_1$, $U_k^* = U_k - U_1$ and $Y_k = \log\left(\sum_{t=1}^{k}(y_t - m_t)\right)^2$.[7] This regression yields

$$\hat{\beta}^* = \frac{\sum_{k=1}^{T} Y_k^* \log k}{\sum_{k=1}^{T} \log^2 k} \quad (3)$$

from which the estimate of the order of summability $\hat{\delta}^* = (\hat{\beta}^* - 1)/2$ is obtained. Inference can be established using confidence intervals constructed from subsample estimation (Politis, Romano and Wolf, 1999), whereby the above procedure is applied to $T - b + 1$ subsamples of length $b = int(\sqrt{T}) + 1$.[8]

Summability is a more general concept than integration, but they are closely related: if a series x_t is integrated of order d, $I(d)$ for $d \geq 0$, then it is also summable of order d, $S(d)$; however, not all $S(d)$ processes are also $I(d)$. Summability analysis thus provides important insights into the time series properties of a variable but in contrast to unit root analysis is not limited to linear processes. In the case of the debt-growth application I pursue here this allows me to investigate the time series properties of squared and cubed debt-to-GDP ratios as well as piecewise linear debt-to-GDP series.

In a second step, Berenguer-Rico and Gonzalo (2014*a*) offer a test to investigate the 'balance' of the empirical relationship, namely the condition that the two sides of the empirical equation have the same order of summability: $S(\delta_y) = S(\delta_z)$ for $z = f(x_t, \theta) = \theta f(x_t)$. Again there is a close analogy with the linear unit root and cointegration case: before cointegration between two or more variables can be tested, it is necessary to establish that these variables possess the same order of integration. Regressing stationary on nonstationary variables – as would be the case if we regressed the per capita GDP *growth rate* on the debt-to-GDP ratio in *levels* – is referred to as an inconsistent regression which leads to invalid inference. However, in the present study I do not test for balance due to an unresolved problem with the testing procedure which invalidates the results.[9]

[7] The deterministic component m_t can be accounted for by the partial mean of y_t, namely $m_t = (1/t)\sum_{j=1}^{t} y_j$ in case of a constant. Given the trending behaviour of my data I focus below on the case of constant and linear trend terms, where partial demeaning of y_t is carried out twice.

[8] I am grateful to a referee who emphasises that the validity of the subsampling procedure has only been shown by simulation.

[9] A referee kindly pointed out that the properties of the balance statistic in a simple model of $y_t = \theta x_t + u_t$, with

It should be noted that the main arguments put forth in this paper are based on the co-summability tests, which do not suffer the same problem.

Finally, the concept of co-summability is tested by investigating the error terms of a candidate specification. In empirical practice, let $\hat{e}_t$ be the least squares residuals from a balanced regression $y_t = \hat{\theta} g(x_t) + \hat{e}_t$, then 'strong co-summability' will imply the order of summability of $\hat{e}_t$ is statistically close to zero, S(0) (Berenguer-Rico and Gonzalo, 2014*a*). Note the analogy to a linear cointegrating relationship where the residuals from a linear regression between I(1) variables will be I(0). The order of summability for $\hat{e}_t$ can be estimated to determine whether a candidate model is co-summable.[10] Inference follows the subsampling approach as in the previous testing procedures and under the null of co-summability the confidence interval includes zero.

3.2 Specifications

I adopt two specifications for nonlinearity in the debt-to-GDP ratio in line with standard approaches in the literature: *first*, in addition to a standard linear model (Model 1) I use polynomial specifications including linear and squared (Model 2) or linear, squared and cubed (Model 3) debt-to-GDP terms (in logarithms) – examples for this specification include Calderon and Fuentes (2013) and Checherita-Westphal and Rother (2012). *Second*, I adopt piecewise linear specifications where the debt-to-GDP ratio (in levels, not logs) is divided into two variables made up of values below and above a specified threshold, which is treated as exogenous (examples for this specification include Kumar and Woo, 2010; Baum, Checherita-Westphal and Rother, 2013; Panizza and Presbitero, 2014).[11] For Great Britain I adopt three threshold values: 90, 70 and 50 percent. For the United States and Japan I can only adopt the 50 percent threshold since even over the full time horizon too few observations are above the other two thresholds: only 12 (Japan: 22) for 70 percent and 6 (Japan: 17) for 90 percent. In Sweden the debt-to-GDP ratio only surpasses the 50 percent threshold in 15 sample years (7% of observations) so that I cannot investigate even a 50%

$x_t \sim S(\delta)$ and $u_t \sim S(\delta)$, are badly affected by θ, especially when $\theta \neq 1$.

[10]The residual series $\hat{e}_t$ will sum to zero by default of the least squares principle if our specification includes an intercept; in practice the estimate for the intercept term is therefore not subtracted when constructing $\hat{e}_t$.

[11]Parts of the literature, including Baum, Checherita-Westphal and Rother (2013), employ threshold regression algorithms where the threshold value is determined endogenously. Extending the co-summability approach in a similar fashion is beyond the scope of this study.

threshold for this country. Note that all of the empirical approaches in the debt-growth literature discussed above are based on models which are linear in parameters but non-linear in the variables – my implementation follows this assumption. Although there are of course alternative transformations (e.g. 'integrable functions' proposed by Park and Phillips, 2001) to model the potential nonlinearity in the debt-growth relationship I restrict myself to the above polynomial and threshold models since these feature in the vast majority of empirical applications – see Panizza and Presbitero (2013) for a recent survey.

The co-summability analysis thus investigates a number of specifications for the debt-growth relationship, inspired by the simple Reinhart and Rogoff (2010*b*) setup. The polynomial specifications are:

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \varepsilon_t \tag{4}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \varepsilon_t \tag{5}$$

$$y_t = \alpha_0 + \varphi t + \phi_1 x_t + \phi_2 x_t^2 + \phi_3 x_t^3 + \varepsilon_t \tag{6}$$

where y is per capita GDP and x is the debt-to-GDP ratio (both in logarithms), α_0 is an intercept, t a linear trend term with parameter φ and ε_t is white noise.

The threshold model specifications are based on

$$\begin{aligned} y_t = {} & \alpha_0 + \varphi t + \theta_1 X_t \times \mathbb{1}(X_t < \text{threshold}) \\ & + \theta_2 X_t \times \mathbb{1}(X_t \geq \text{threshold}) + \varepsilon_t \end{aligned} \tag{7}$$

where $\mathbb{1}(X_t < \text{threshold})$ is an indicator function which is 1 for the debt-to-GDP ratio X_t below the threshold and 0 otherwise – similarly for $\mathbb{1}(X_t \geq \text{threshold})$ at and above the threshold.

I investigate the evidence for long-run equilibrium relationships between debt burden and per capita GDP *levels* – since the focus of the applied literature is on the *long-run* relationship I adopt the levels variable for income, rather than its growth rate. The popularity of the 'growth' specification in the cross-country empirical literature is justified by the presence of the lagged level of per capita GDP as additional regressor (as is the case for the 'debt-growth' analysis of Kumar and Woo, 2010; Cecchetti, Mohanty and Zampolli, 2011; Checherita-Westphal and Rother, 2012; Baum, Checherita-Westphal and Rother, 2013, among others). This quasi-error correction specification provides estimates for a long-run *levels* relationship although researchers frequently refer to this type of specification as a 'growth' equation (see Eberhardt and Teal, 2011).

In addition to the analysis for the full time horizon I investigate co-summability in the four OECD countries using a window of sixty years, which is moved along the time horizon from the 1800s to 2010. The purpose of this exercise is to provide both an indication of possible changes in the long-run debt-growth relationship over time as well as to safeguard the analysis from undue impact of severe shocks such as the two world wars or changes in the definition or the debt variable.[12] Due to the nature of the data this approach is only feasible for the polynomial specifications: as highlighted by Chinn (2012) in his review of Reinhart and Rogoff (2011) there are comparatively few episodes in developed economies where the debt-to-GDP ratio exceeds 90% and I can therefore not implement the moving window for the piecewise linear specification. Since this rolling window analysis represents a form of data mining I adjust the confidence intervals (CI) for all estimates following a standard Bonferroni correction, whereby $\text{CI}^* = (1 - \alpha/m)$ for the conventional confidence level $1 - \alpha$ (I adopt $\alpha = .05$) and the number of sub-samples tested m (varies from 80 for Japan to 152 for the US, Great Britain and Sweden). In practice this makes the confidence intervals *much wider*, thus representing a more conservative approach to rejecting the null hypothesis of co-summability. A number of additional robustness checks are carried out, for which the motivation, approach, and results are presented in an Online Appendix. The focus of these robustness checks is on (i) a diverse sample of 23 additional economies (including some developing countries); (ii) a reverse specification with debt-to-GDP ratio (in logs) as the dependent variable for models including the (log of) per capita GDP, and its squared and cubed polynomial terms as regressors; (iii) economic theory-based specifications which add a number of determinants of growth as favoured by the cross-country growth literature to the model.

[12] The data used here refer to central government debt, which excludes any debt from local government, as opposed to general government debt. As shown in the work of Dippelsman, Dziobek and Mangas (2012), the quantitative implications of this choice of variable can be stark. However, in the absence of any general government debt data over the long time horizon the analysis here is forced to employ the conceptually inferior 'central government' measures – this choice is however aligned with the analysis in Reinhart and Rogoff (2010*b*).

4 Data

I use annual per capita GDP (in 1990 Geary-Khamis $) from an updated version (Bolt and van Zanden, 2013) of the series compiled by Maddison (2010). I match these data to information on the gross government debt-to-GDP ratio (in percent) from Reinhart and Rogoff (2009). The debt figures refer to total gross central government debt, comprising domestic and external debt (see Online Appendix for exceptions). Data coverage differs across countries: for the US, Britain and Sweden data series start in 1800, for Japan in 1872 – all series end in 2010.

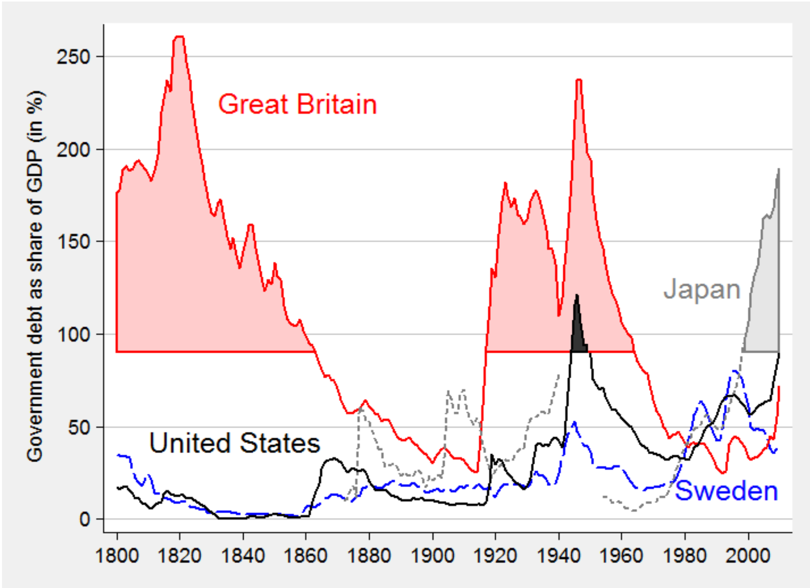
Descriptive statistics for these four countries are presented in the Online Appendix, where I also plot the levels and first differences of the per capita GDP and debt-to-GDP ratio variables (in logs). Although my summability analysis provides insights into the time series properties of these data I also carry out a number of unit root tests to illustrate the difference in order of integration between the per capita GDP growth rate and the debt-to-GDP ratio in levels which rules out the existence of any long-run relationship (cointegration, co-summability) between these two variable series.

In the Online Appendix data from a further 23 countries using the same sources are employed to carry out summability and co-summability tests. Here countries were included in the sample provided their per capita GDP and debt-to-GDP ratio series extended back to 1900 or earlier.

Extended empirical models analysed in an Online Appendix incorporate inflation and schooling data primarily taken from the Clio Infra project at the International Institute of Social History, population data from the original Maddison (2010) dataset, investment and additional debt data from Maddison (1992), Mitchell (2007*a*,*b*), and the World Bank *World Development Indicators* as well as a number of other sources (for details see Online Appendix).

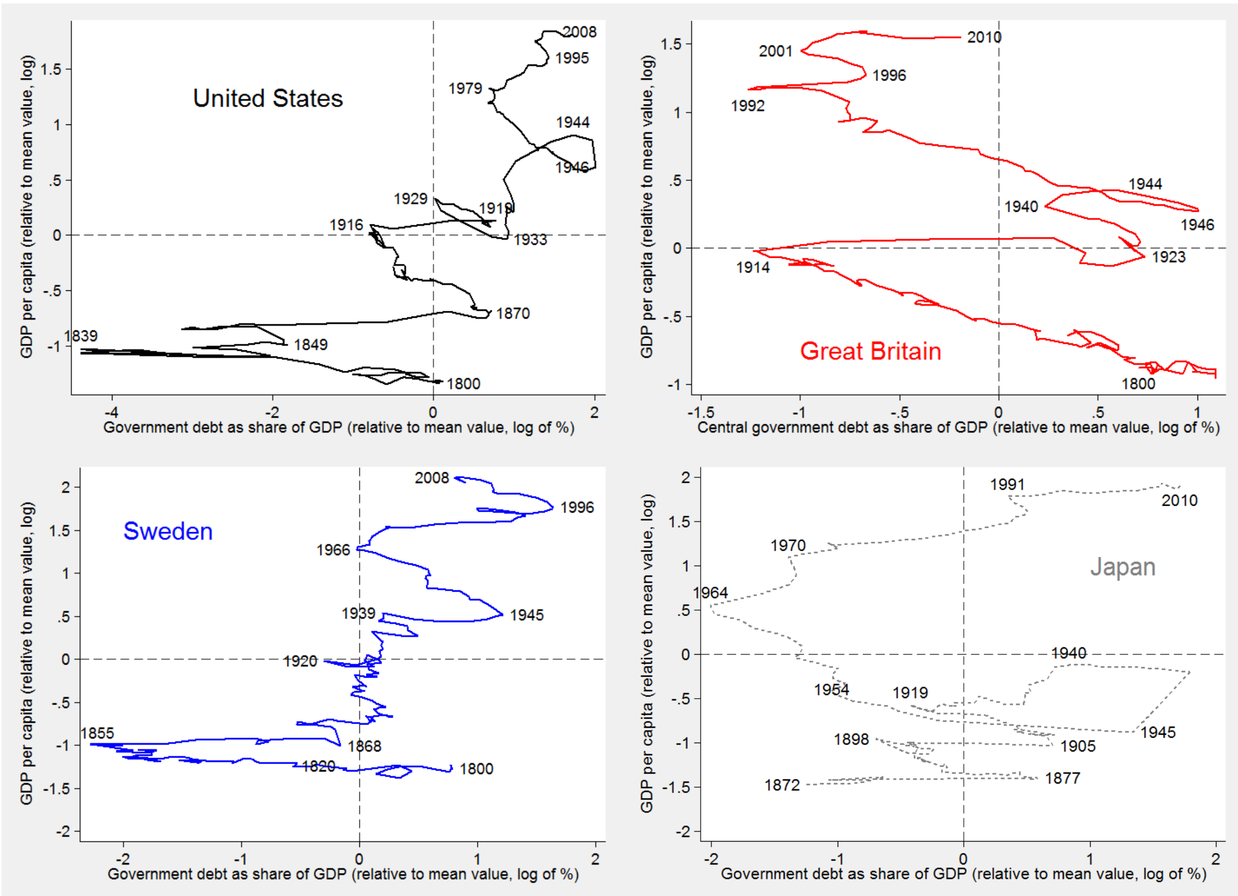
Figure 1 charts the evolution of the debt-to-GDP ratio for the four economies, where in the spirit of Reinhart and Rogoff (2010*b*) I highlight periods with debt burden in excess of 90% of GDP. While the four time series all display idiosyncracies, it is nevertheless notable how similar in particular the patterns for British and American debt-to-GDP ratios are over much of the 20th century, albeit with substantially higher debt in the former. Britain is also the only economy studied which experienced sustained periods of debt-to-GDP above 90%.

Figure 1: Evolution of Debt/GDP ratios



Notes: The shaded areas represent the periods where debt/GDP exceeded 90%.

Figure 2: Debt Ratio and Income Per Capita



Notes: Debt ratios and per capita GDP series (both in logarithms) are presented in deviation from their country-specific time-series means (within transformation).

In Figure 2 I plot the debt-income relationship in each of the four countries, taking variables in deviation from the country-specific time-series mean. In all four economies the most significant turning points for the debt-growth nexus were marked by the Great War of 1914-18, the Great Recession of the late 1920s and World War II.

5 Empirical Results

5.1 Main results: Order of Summability

Table 1 provides estimates of the order of summability for all model variables, including polynomial as well as threshold terms for debt. None of the confidence intervals for tests on per capita GDP *levels* or any of the debt variables include zero, thus rejecting the null of summability of order zero. The estimated order of summability for the per capita GDP *growth rates* in contrast is always very close to zero. For the linear terms of per capita GDP and the debt-to-GDP ratio (in logs or levels) and their growth rates these results are perfectly in line with unit root and stationarity test results presented in the Online Appendix, where I establish stationary growth rates and nonstationary levels series (whether in logarithms or not).

These findings highlight the significant persistence in the data and provide a strong motivation for the concerns over time series properties I argue are of primary importance when analysing the long-run debt-growth nexus. In analogy to integrated data, we run the risk of spurious results in any regressions containing these variables unless we can confirm our empirical models as balanced and co-summable. Note that with the exception of the study by Balassone, Francese and Pace (2011) on Italy none of the papers in this literature show concern for time series properties of the data.

Table 1: Estimated Order of Summability

Country	Start & End Year		Obs	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	211	ln(GDP pc)	0.652	1.490	2.329
				Δln(GDP pc)	-0.524	0.066	0.657
				ln(Debt/GDP)	0.551	1.082	1.613
				ln(Debt/GDP) squared	0.383	0.860	1.336
				ln(Debt/GDP) cubed	0.404	0.993	1.582
			168	Debt/GDP $<$ 50%	0.313	0.825	1.337
			43	Debt/GDP $\geq$ 50%	0.691	1.409	2.127
GBR	1800	2010	211	ln(GDP pc)	0.802	1.839	2.877
				Δln(GDP pc)	-0.452	0.130	0.712
				ln(Debt/GDP)	0.540	0.967	1.393
				ln(Debt/GDP) squared	0.509	0.948	1.386
				ln(Debt/GDP) cubed	0.475	0.931	1.387
			100	Debt/GDP $<$ 90%	0.511	1.062	1.613
			111	Debt/GDP $\geq$ 90%	0.405	0.936	1.467
			86	Debt/GDP $<$ 70%	0.428	1.200	1.972
			125	Debt/GDP $\geq$ 70%	0.465	0.923	1.381
			64	Debt/GDP $<$ 50%	0.447	1.068	1.689
			147	Debt/GDP $\geq$ 50%	0.459	0.898	1.336
SWE	1800	2010	211	ln(GDP pc)	0.495	0.897	1.298
				Δln(GDP pc)	-1.110	-0.378	0.355
				ln(Debt/GDP)	0.645	1.624	2.603
				ln(Debt/GDP) squared	0.704	1.577	2.451
				ln(Debt/GDP) cubed	0.677	1.538	2.399
JPN	1872	2010	139	ln(GDP pc)	0.987	2.390	3.792
				Δln(GDP pc)	-0.692	-0.004	0.683
				ln(Debt/GDP)	0.427	1.091	1.755
				ln(Debt/GDP) squared	0.433	1.101	1.769
				ln(Debt/GDP) cubed	0.410	1.114	1.819
			85	Debt/GDP $<$ 50%	0.543	1.282	2.022
			54	Debt/GDP $\geq$ 50%	0.192	1.025	1.858

Notes: CI low and up indicate the 95% confidence interval for the summability estimate $S(\delta)$ constructed from subsampling – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

5.2 Main results: Co-Summability

Table 2 provides results from co-summability tests using per capita GDP *levels* as dependent variable. Co-summability is rejected in *all* countries and specifications – residuals from these models were not found to be summable of order zero, S(0). Note that the rejection of co-summability is by no means marginal, with all confidence intervals some distance away from zero. The fact that subsampling confidence intervals are at times very wide is a further strong signal for misspecification. These findings imply that from a long-run perspective per capita income and the debt-to-GDP ratio do not move together, *precluding any causal relationship between these variables*.[13]

Table 2: Co-Summability – ln(GDP pc) specifications

	Start	End		obs	Nonlinearity	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1870	2010		141	-	0.467	1.049	1.631	$S(\delta_{\hat{e}_t}) \neq 0$
			$b =$	13	Debt/GDP squared	0.277	0.943	1.609	$S(\delta_{\hat{e}_t}) \neq 0$
			$M =$	129	Debt/GDP cubed	0.351	0.900	1.449	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 50%	0.526	1.124	1.721	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1830	2010		181	-	0.660	1.194	1.728	$S(\delta_{\hat{e}_t}) \neq 0$
			$b =$	14	Debt/GDP squared	0.699	1.196	1.693	$S(\delta_{\hat{e}_t}) \neq 0$
			$M =$	168	Debt/GDP cubed	0.702	1.196	1.689	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 90%	0.653	1.208	1.763	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 70%	0.656	1.224	1.791	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 50%	0.668	1.284	1.899	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1820	2010		191	-	0.777	1.546	2.314	$S(\delta_{\hat{e}_t}) \neq 0$
			$b =$	15	Debt/GDP squared	0.658	1.602	2.546	$S(\delta_{\hat{e}_t}) \neq 0$
			$M =$	177	Debt/GDP cubed	0.697	1.598	2.499	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010		139	-	0.580	1.128	1.676	$S(\delta_{\hat{e}_t}) \neq 0$
			$b =$	13	Debt/GDP squared	0.236	0.873	1.511	$S(\delta_{\hat{e}_t}) \neq 0$
			$M =$	127	Debt/GDP cubed	0.181	0.821	1.460	$S(\delta_{\hat{e}_t}) \neq 0$
					Threshold 50%	0.857	2.056	3.256	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant only; additional trends do not qualitatively change the results). $\hat{\delta}_{\hat{e}_t} \neq (=)0$ implies that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).

5.3 Results from Subsample Analysis

Subsample analysis yields three sets of results: (i) country-specific time-varying co-summability statistics for the entire 152 subsamples (80 for Japan) of sixty years, which I present in graphical form; (ii) comparison of the co-summability subsample results for the United States, Great Britain and Sweden, again in graphical form – this is intended to uncover patterns of commonality and

[13] In additional work presented in the Online Appendix I investigate a 'reversed' model for debt with linear and polynomial terms of per capita GDP as the regressors and similarly find precious little evidence for a long-run equilibrium relationship.

difference in the equilibrium relationship across countries; (iii) co-summability statistics for the post-WWII period as well as results omitting the most recent years covering the global financial crisis (2008-2010).

Graphical results for the sub-sample analysis of co-summability, including Bonferroni-adjusted confidence intervals, are presented in Figure 3.[14] In each plot the end-year of the sixty-year window of analysis is marked on the x-axis and shading indicates the Bonferroni-adjusted 95% confidence intervals – due to different data availability this time dimension of the plots differ for Japan. Note first that across all models and countries the confidence intervals are fairly large, typically from 0 to 2 or larger. Second, while (Japan aside) in each country the share of samples which satisfy co-summability is typically above 50%,[15] this data property does not appear to be satisfied consistently over longer stretches of time, but instead appears sporadically. Both of these findings provide a strong signal of misspecification and thus echo the full sample results presented above.

In Table 3 I compare the subsample periods for which the sixty-year data series constituted co-summable specifications in the data for the US, Great Britain and Sweden: Panel A refers to the linear model (Model 1), Panels B and C to the polynomial specifications with (additional) squared terms and squared and cubed debt terms, respectively (Models 2 and 3). For each country a shaded cell indicates the sixty-year subsample ending in the year specified constitutes a co-summable specification, while the intensity of the shading indicates whether this property occurs in one (lightest), two (intermediate) or all three (darkest) countries. Japan is excluded in this graphical analysis since the difference in available time series data would necessitate different shading between earlier (excluding Japan) and later periods (including Japan) which would make a mockery of my attempts to use graphs to illustrate commonality. I begin by focusing on those 'episodes' of long-run co-movement when the tests for *all three countries* find co-summability: in all models clusters of such episodes can be found in the 1860s (thus for the series starting in the early 1800s), the 1890s-1910s (1830s–1850s), and the 1950s and 1960s (1890s–1900s, incorporating both World Wars). Thereafter isolated episodes pop up in the 1970s. The most

[14]Not adopting the Bonferroni adjustment would lead to significantly *narrower* confidence intervals, which in all cases would yield the same or a stronger qualitative result of limited evidence for a long-run equilibrium relationship between debt and income.

[15]The overall share of samples which satisfy co-summability is as follows: USA 50%, GBR 52%, SWE 49%, JPN 14% (Model 1); USA 56%, GBR 64%, SWE 64%, JPN 17% (Model 2); USA 59%, GBR 73%, SWE 64%, JPN 28% (Model 3). Results are qualitatively similar if I adopt a longer (70-year) window instead.

recent episodes occurred in the early 2000s, which incorporate sample years during WWII and its immediate aftermath. Taken together these various episodes account for 28% of all subsamples across the three specifications.[16] Note that the years of the global financial crisis (2008-10) do not form part of this cluster of co-summable episodes in all three countries.

Referring back to Figure 2 it can be seen that the first of these clusters, covering subsamples ending in the 1860s, occurred when all three countries substantially reduced their country-specific debt-burden (movement to the left in Figure 2) albeit with comparatively modest increase in growth in the US and Sweden (relatively flat line plots). No such pattern is revealed for the second cluster for subsamples ending in the 1890s and 1900s, while the third cluster with end years in the 1950s and 60s occurred when all three countries shifted from a relative debt build-up in years prior to and during WWII to significant debt reduction thereafter, whereby the latter period also represented a return to steady economic growth. The final cluster in the early 2000s again does not reveal any systematic patterns in the evolution of debt burden and growth across these three economies.

Inbetween these episodes there are stretches where two countries have co-summable specifications (around 31-38% of subsamples in each model), although these are often clustered around the episodes just described. The remainder of subsamples is made up of single country episodes (20-34% in each model) and subsamples with no co-summability in any country (7-16% in each model).

Table 4 then zooms in on the post-WWII period which forms the focal point for virtually all existing empirical studies on the debt-growth nexus. Here we find some evidence for co-summability in nonlinear specifications (these sub-samples are shaded in grey), especially for the model including a cubed term. Note however that the confidence intervals for the overwhelming majority of these results are very large (indicated with the darker shading), such that they include 1 and at times even 2: a large confidence interval is indicative of serious misspecification and these findings of co-summability should thus be treated with caution.

[16]In Model 1 they make up 17% of all subsamples, in Models 2 and 3 30% and 36% respectively.

Figure 3: Co-Summability Testing (Sub-Samples)

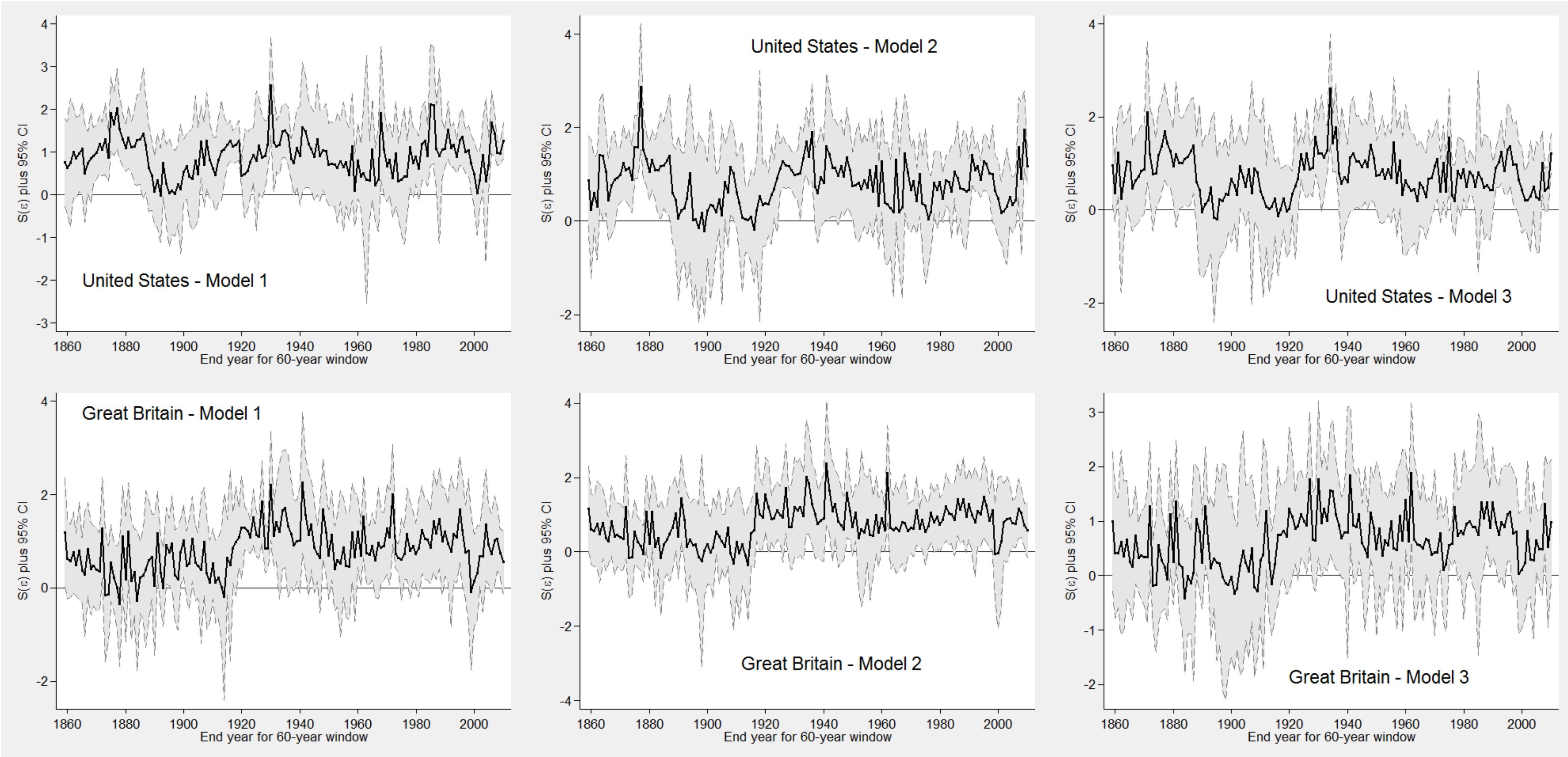
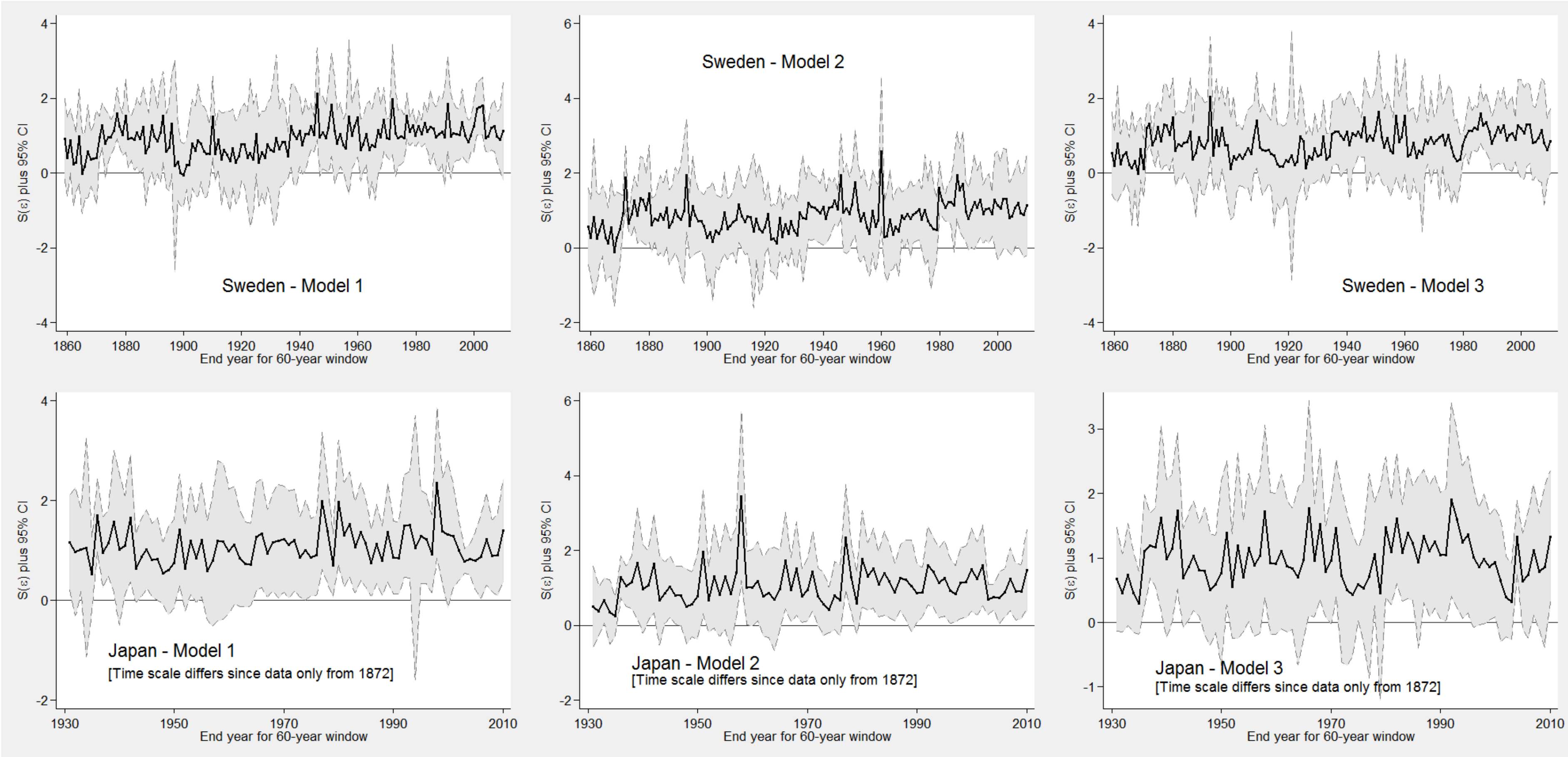


Figure continued on the following page

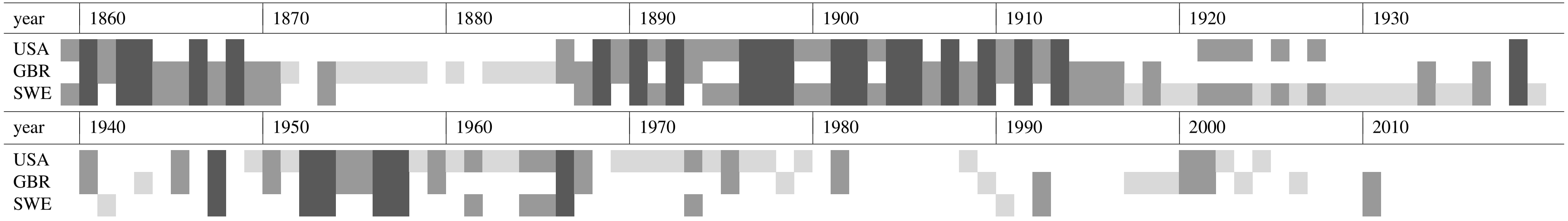
Figure 3: Co-Summability Testing (Sub-Samples) continued



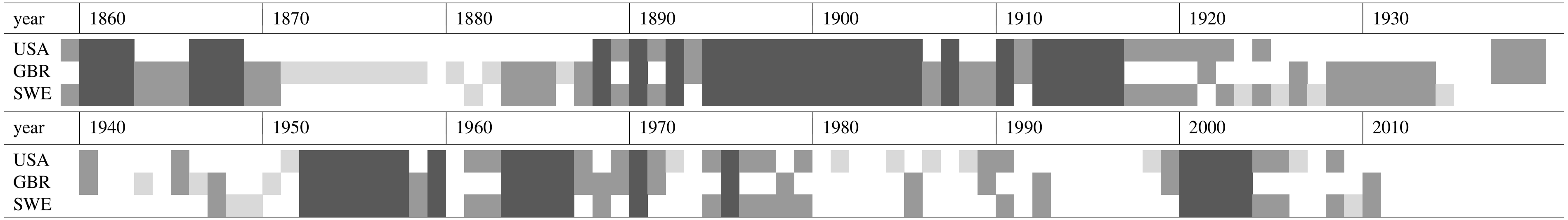
Notes: The shaded areas represent the Bonferroni-corrected 95% Confidence Intervals for the Co-Summability statistic computed in a moving window of 60-year time periods. The solid black line represents the computed Co-Summability statistic. I allow for an intercept in the co-summability analysis. The coverage of the data differs across countries: for the US, Great Britain and Sweden I have data from 1800-2010 (152 subsamples), for Japan from 1872-2010 (80 subsamples). Model 1 refers to a specification with linear debt terms only, Model 2 to a specification with linear and quadratic debt terms, Model 3 further includes a cubed debt term.

Table 3: Co-Summability – Cross-Country Comparison

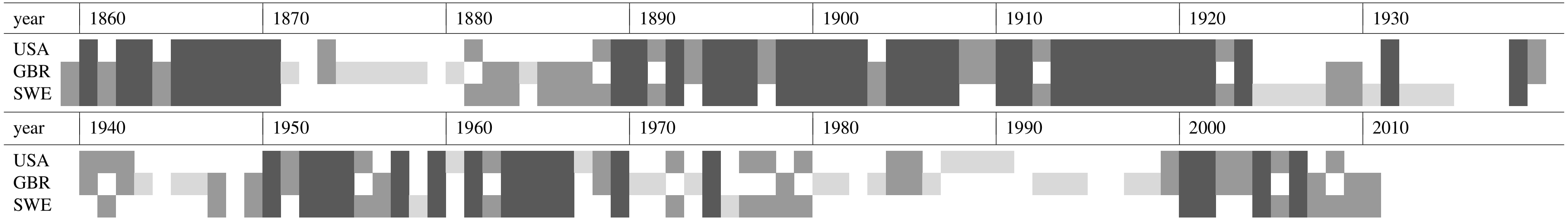
PANEL A: Linear Specification (Model 1)



PANEL B: Specification with squared debt (Model 2)



PANEL C: Specification with squared and cubed debt (Model 3)



Notes: Increasing shading indicates the number (0-3) of countries for which the sixty-year subsample ending in the year indicated has a co-summable specification.

These robustness checks provide a number of important insights: *first*, there is no overwhelming evidence that these full sample results are severely distorted by global shocks or structural breaks in the long-run debt-growth relationship, given that a very considerable share of subsamples were found to not to be co-summable across all countries and specifications. *Second*, having said that my results point to a distinct possibility that certain countries experienced linear or nonlinear co-movement between debt and income during certain periods of time over the past two centuries, although seemingly much less so during the 20th century.

6 Concluding Remarks

This study took an alternative approach to investigating the presence of nonlinearities in the long-run equilibrium relation between public debt and growth. Empirical results for four OECD countries using data from 1800 to 2010 and the various robustness checks carried out provide limited evidence for nonlinear, or indeed linear, long-run relationships between these variables. There are however certain subperiods over this long time horizon for which tests confirm co-movement between debt and income. The timing of these subperiods of co-movement frequently appears to differ across countries. These findings are not narrowly confined to the four OECD economies studied in detail but seem to have much wider validity, and further are not an artefact of the simple model specification adopted: I investigated summability and co-summability in a sample of 23 additional countries (including some developing countries), and furthermore studied a number of theory-based extended specifications for the four OECD economies; results in the Online Appendix provide strong support for the findings presented above.

Table 4: Co-Summability – ln(GDP pc) specifications (sub-sample results for post-WWII period)

			United States				United Kingdom			
M	start	end	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
1	1946	2005	0.051	0.699	1.347	$S(\delta_{\hat{e}_t}) \neq 0$	0.080	0.936	1.792	$S(\delta_{\hat{e}_t}) \neq 0$
	1947	2006	0.965	1.697	2.428	$S(\delta_{\hat{e}_t}) \neq 0$	-0.126	0.725	1.577	$S(\delta_{\hat{e}_t}) = 0$
	1948	2007	0.886	1.435	1.984	$S(\delta_{\hat{e}_t}) \neq 0$	0.072	1.024	1.977	$S(\delta_{\hat{e}_t}) \neq 0$
	1949	2008	0.698	0.987	1.277	$S(\delta_{\hat{e}_t}) \neq 0$	0.360	1.051	1.742	$S(\delta_{\hat{e}_t}) \neq 0$
	1950	2009	0.704	0.964	1.225	$S(\delta_{\hat{e}_t}) \neq 0$	0.220	0.738	1.256	$S(\delta_{\hat{e}_t}) \neq 0$
	1951	2010	0.836	1.263	1.690	$S(\delta_{\hat{e}_t}) \neq 0$	-0.145	0.551	1.247	$S(\delta_{\hat{e}_t}) = 0$
2	1946	2005	-0.250	0.347	0.944	$S(\delta_{\hat{e}_t}) = 0$	0.158	0.780	1.402	$S(\delta_{\hat{e}_t}) \neq 0$
	1947	2006	-0.128	0.444	1.016	$S(\delta_{\hat{e}_t}) = 0$	0.101	0.765	1.428	$S(\delta_{\hat{e}_t}) \neq 0$
	1948	2007	0.504	1.573	2.641	$S(\delta_{\hat{e}_t}) \neq 0$	0.375	1.170	1.966	$S(\delta_{\hat{e}_t}) \neq 0$
	1949	2008	-0.741	0.903	2.548	$S(\delta_{\hat{e}_t}) = 0$	0.385	1.048	1.711	$S(\delta_{\hat{e}_t}) \neq 0$
	1950	2009	1.132	1.954	2.777	$S(\delta_{\hat{e}_t}) \neq 0$	0.130	0.709	1.289	$S(\delta_{\hat{e}_t}) \neq 0$
	1951	2010	0.777	1.169	1.561	$S(\delta_{\hat{e}_t}) \neq 0$	-0.167	0.585	1.336	$S(\delta_{\hat{e}_t}) = 0$
3	1946	2005	-0.924	0.277	1.478	$S(\delta_{\hat{e}_t}) = 0$	0.057	0.648	1.238	$S(\delta_{\hat{e}_t}) \neq 0$
	1947	2006	-0.913	0.233	1.379	$S(\delta_{\hat{e}_t}) = 0$	-0.168	0.490	1.148	$S(\delta_{\hat{e}_t}) = 0$
	1948	2007	0.337	1.017	1.697	$S(\delta_{\hat{e}_t}) \neq 0$	-0.361	0.488	1.336	$S(\delta_{\hat{e}_t}) = 0$
	1949	2008	-0.066	0.411	0.887	$S(\delta_{\hat{e}_t}) = 0$	0.433	1.319	2.205	$S(\delta_{\hat{e}_t}) \neq 0$
	1950	2009	0.008	0.482	0.956	$S(\delta_{\hat{e}_t}) \neq 0$	-0.969	0.525	2.018	$S(\delta_{\hat{e}_t}) = 0$
	1951	2010	0.790	1.217	1.644	$S(\delta_{\hat{e}_t}) \neq 0$	-0.159	0.982	2.123	$S(\delta_{\hat{e}_t}) = 0$

			Sweden				Japan			
M	start	end	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
1	1946	2005	0.558	0.974	1.391	$S(\delta_{\hat{e}_t}) \neq 0$	0.205	0.794	1.383	$S(\delta_{\hat{e}_t}) \neq 0$
	1947	2006	0.625	1.223	1.821	$S(\delta_{\hat{e}_t}) \neq 0$	0.039	0.879	1.720	$S(\delta_{\hat{e}_t}) \neq 0$
	1948	2007	0.575	1.265	1.955	$S(\delta_{\hat{e}_t}) \neq 0$	0.311	1.234	2.156	$S(\delta_{\hat{e}_t}) \neq 0$
	1949	2008	0.311	0.984	1.657	$S(\delta_{\hat{e}_t}) \neq 0$	0.173	0.893	1.613	$S(\delta_{\hat{e}_t}) \neq 0$
	1950	2009	0.075	0.899	1.722	$S(\delta_{\hat{e}_t}) \neq 0$	0.099	0.913	1.726	$S(\delta_{\hat{e}_t}) \neq 0$
	1951	2010	-0.174	1.127	2.428	$S(\delta_{\hat{e}_t}) = 0$	0.380	1.402	2.423	$S(\delta_{\hat{e}_t}) \neq 0$
2	1946	2005	-0.205	0.847	1.900	$S(\delta_{\hat{e}_t}) = 0$	0.041	0.733	1.425	$S(\delta_{\hat{e}_t}) \neq 0$
	1947	2006	0.039	1.071	2.103	$S(\delta_{\hat{e}_t}) \neq 0$	0.225	0.876	1.526	$S(\delta_{\hat{e}_t}) \neq 0$
	1948	2007	0.052	1.203	2.354	$S(\delta_{\hat{e}_t}) \neq 0$	0.396	1.237	2.078	$S(\delta_{\hat{e}_t}) \neq 0$
	1949	2008	-0.074	0.933	1.941	$S(\delta_{\hat{e}_t}) = 0$	0.145	0.899	1.653	$S(\delta_{\hat{e}_t}) \neq 0$
	1950	2009	-0.260	0.881	2.023	$S(\delta_{\hat{e}_t}) = 0$	0.185	0.905	1.626	$S(\delta_{\hat{e}_t}) \neq 0$
	1951	2010	-0.197	1.137	2.471	$S(\delta_{\hat{e}_t}) = 0$	0.405	1.482	2.560	$S(\delta_{\hat{e}_t}) \neq 0$
3	1946	2005	-0.092	0.823	1.738	$S(\delta_{\hat{e}_t}) = 0$	-0.095	0.643	1.381	$S(\delta_{\hat{e}_t}) = 0$
	1947	2006	-0.200	0.947	2.094	$S(\delta_{\hat{e}_t}) = 0$	-0.240	0.738	1.717	$S(\delta_{\hat{e}_t}) = 0$
	1948	2007	-0.241	1.146	2.532	$S(\delta_{\hat{e}_t}) = 0$	0.106	1.120	2.133	$S(\delta_{\hat{e}_t}) \neq 0$
	1949	2008	-0.853	0.804	2.460	$S(\delta_{\hat{e}_t}) = 0$	-0.104	0.786	1.677	$S(\delta_{\hat{e}_t}) = 0$
	1950	2009	-0.201	0.621	1.444	$S(\delta_{\hat{e}_t}) = 0$	-0.388	0.858	2.103	$S(\delta_{\hat{e}_t}) = 0$
	1951	2010	-0.020	0.853	1.727	$S(\delta_{\hat{e}_t}) = 0$	0.317	1.332	2.347	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: This table presents results for sub-sample co-summability testing. Model (M) 1-3 refer to the following specifications: 1 – linear, 2 – linear and squared, 3 – linear, squared and cubed debt/GDP terms (in logs). In each case I report statistics from the sixty-year samples, with start and end years as indicated in the table, focusing on the period after 1945. Shaded cells indicate sub-samples where the co-summability confidence interval includes zero – note however that these confidence intervals are at times very large, such that they further include 1 and at times even 2 (the latter options are represented by darker shading). See Table 2 for all other details.

It is important to emphasise that this study *does not and cannot* address causality from high(er) debt to low(er) growth as has been the focus in most of the empirical work on this topic. This is by no means a shortcoming of the approach taken. Instead, it highlights a central inconsistency in the empirical analysis of nonlinearities in the debt-growth relationship in the existing literature: in order to establish a long-run causal relationship from debt to growth, it is necessary to *first* establish a long-run equilibrium relationship. This study documents the difficulties for establishing the latter using standard empirical specifications adopted in the literature when variable series are integrated. Once these difficulties are addressed, I find no evidence for a long-run equilibrium relation in the data for four OECD countries. Various robustness checks provide assurance that this finding is not an artefact of sample selection. Since a long-run equilibrium relationship represents a pre-requisite for any long-run causality between variables my analysis by necessity stops at this point.

The results presented in this paper undermine some of the popular conclusions for this politically-charged issue which represent fiscal adjustment as a necessity for *long-run* economic stability and sustainability. I do not claim that a high debt burden is a matter of no concern for policymakers or that in the short-run debt may not be detrimental to growth. Instead, I highlight the absence of evidence for nonlinearities such as the popular 90% debt-to-GDP threshold or polynomial specifications in the *long-run* relationship with growth and development, which has been the explicit focus of the empirical literature I cite and review.

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Online Appendix

A.1 Data sources and construction

Per capita income and debt

In the empirical analysis I employ per capita GDP in 1990 constant international dollars (in logarithms) and the percentage debt-to-GDP ratio (also in logarithms), where debt is generally gross central (external and domestic) government debt. The raw data for the analysis of four OECD countries is taken from the Excel spreadsheets available on the following websites: for data on debt www.reinhartandrogoff.com – the Reinhart and Rogoff (2009) companion website; and for data on per capita income (as well as population data) www.ggdc.net/maddison – the Maddison Project website at the University of Groningen.

A number of comments regarding these data as well as changes and/or adjustments are provided in the following:

- For Great Britain the debt series refer to net rather than gross central (external and domestic) government debt.
- The 'New' Maddison data provides two values for Great Britain's per capita GDP in 1851 – 2,330 and 2,718 – since this is where two data series come together: up to 1851 the estimates are taken from Van Zanden (2001), from 1851 onwards from the original Maddison (2010) estimates. I pick the arithmetic mean of the two values.
- Data on the debt-to-GDP ratio for Japan between 1941 and 1951 are computed from outstanding debt reported in Statistics Japan's *Historical Statistics of Japan*, available at www.stat.go.jp/english/data/chouki/05.htm. The 1952 and 1953 figures are computed from debt and GDP numbers in the excel spreadsheets marked 'RR' provided by Herndon, Ash and Pollin (2014). These are available at www.peri.umass.edu/236/hash/31e2ff374b6377b2ddec04deaa6388b1/publication/566/. Japanese GDP figures for 1941-51 are taken from Mitchell (2007*a*).
- I interpolated the debt-GDP ratio for Japan in 1882 where only a single observation was

missing in the Reinhart and Rogoff data. Similarly for the GDP series in 1945.

The analysis of time series of debt and per capita income in 27 economies is based on the same data sources. Below I discuss any changes made to variables in that sample:

- I interpolated the debt-to-GDP ratio in 3 cases where only a single observation was missing or (in case of India) where the recorded value was not credible (zero debt): Argentina 1866, India 1947, Peru 1960.
- For Brazil I chose debt data starting from 1889 since prior to this date the series covered only external debt. For Argentina, Italy, the Netherlands and New Zealand the debt series represent general rather than central government debt.

Additional Variables

Inflation I use inflation data from the Clio Infra project at the *International Institute of Social History* which reports the annual percentage change. Excel spreadsheets are available for download at www.clio-infra.eu. Downloads automatically include very detailed information on the original sources of the data. The following adjustments were made: for *Chile* in 1914 (two values provided), I took the arithmetic mean. Data for *Austria* in 1915 was missing so I linearly interpolated, similarly for *Australia* in 1911 and 1912, and *New Zealand* in 2010 (extrapolated).

Population The original Maddison (2010) database provides data on mid-year population (in thousands). My analysis employs the population growth rate. For all population series I extrapolate the value for 2010 from the Maddison data to maintain the integrity of the data series (alternative sources, e.g. the Penn World Table version 8.0, gave marginally different figures for the last years leading up to 2010 than the Maddison data).

- Population data for *Sweden* during 1800-1819 is taken from Schön and Krantz (2012). Excel spreadsheets are available from www.ekh.lu.se/en/research/economic_history_data/shna1560-2010.
- Population data for the *United States* during 1800–1819 is taken from the US Bureau of the Census, *Historical Statistics of the United States, Colonial Times to 1970*, Bicentennial Edi-

tion, Part 2, Washington, D.C., 1975. A pdf (Series A 6-8) is available from www.census.gov/prod/www/statistical_abstract.html.

– Population data for the *United Kingdom* 1800–1819 is taken from Mitchell (1971), page 8.

– Population data for *Argentina* in 1869 and 1895 is taken from Argentina's Instituto Nacional de Estadistica y Censos (indec), www.indec.mecon.ar/. Data for 1870–1894 and 1896–1899 are linearly interpolated.

Investment This is the investment-to-GDP ratio in percent (in the empirical analysis further in logarithms). The main source for these data are the tables available on the GGDC Maddison website, which report Maddison's (1992) domestic capital formation in percent of GDP. Below I indicate additional and alternative sources for these data ('Maddison' here refers to the 1992 *SJE* data):

– *Argentina* 1900–2010 is taken from the Montevideo-Oxford Latin American Economic History Data Base (MOxLAD) available at www.lac.ox.ac.uk/moxlad-database.

– *Australia* 1861–1869 computed from Mitchell (2007*a*); 1870–1989 from Maddison; for the remainder of the series I use the World Bank World Development Indicators (WDI) gross fixed capital formation (GFCF) series.

– *Brazil* 1900–2010 from MOxLAD.

– *Canada* 1870–1988 from Maddison; remainder from WDI.

– *Chile* 1900–2010 from MOxLAD.

– *Colombia* 1900–2010 from MOxLAD.

– *Denmark* 1850–1945 from Jones and Obstfeld (1997); 1946–1969 from Mitchell (2007*b*); remainder from WDI.

– *France* 1870–1988 from Maddison; 1850–1869, 1919–21, and 1939 from Jones and Obstfeld (1997); post-1939 from WDI.

– *Germany* 1925–1988 from Maddison; 1850–1913 from Jones and Obstfeld (1997); post-1988 from WDI.

- *India* 1870–1987 from Maddison; remainder from WDI.
- *Japan* 1885–1988 from Maddison, where I linearly interpolated for 1945; post-1988 from WDI.
- *Netherlands* 1921–1988 from Maddison (with gaps), for 1807–1913 I compute the investment/GDP ratio from GDP at current prices and total current gross fixed capital formation from the *National Accounts of the Netherlands 1800–1913*, provided by the Netherlands Research Institute and Graduate School on Economic and Social History at Data Archiving and Networked Services (DANS), available online at nationalaccounts.niwi.knaw.nl/start.htm; post-1988 from WDI.
- *Norway* 1830–2010 from Norges Bank *Historical Monetary Statistics – GDP and its components from 1830 to 2010* available at www.norges-bank.no/en/Statistics/.
- *Sweden* 1800–2000 is taken from the *Historical National Accounts for Sweden 1800–2000* available at www.ekh.lu.se/en/research/economic_history_data/shna1560-2010; post-2000 from WDI.
- *United Kingdom*: 1850–1945 from Jones and Obstfeld (1997); thereafter from Maddison (to 1988); post-1945 from WDI.
- *United States*: 1870–1986 from Maddison; post-1986 from WDI.

Human Capital Average years of education in the population is taken from the Excel spreadsheets provided by the Clio Infra project at www.clio-infra.eu.

A.2 Descriptives and Data Properties

Descriptive statistics for the four OECD countries analysed in the maintext are presented in Table A-1. Table A-2 reports results from unit root tests for all linear processes. This indicates that growth rates of per capita GDP or debt-to-GDP are stationary, while the levels of these variables are nonstationary. Figure A-1 charts the log levels and growth rates of per capita GDP (left column) and the debt-to-GDP ratio (right column) in the four sample countries.

Table A-1: Descriptive Statistics

		Start	End	Obs	Mean	Median	St.Dev.	Min	Max
USA	ln(GDP pc)	1800	2010	211	8.510	8.425	0.996	7.159	10.363
	Δln(GDP pc)			210	0.015	0.017	0.047	-0.241	0.171
	ln(Debt/GDP)			211	2.667	2.958	1.629	-5.878	4.798
	ln(Debt/GDP)^2			211	9.754	9.090	6.397	0.010	34.552
	ln(Debt/GDP)^3			211	31.026	25.870	36.127	-203.104	110.445
GBR	ln(GDP pc)	1800	2010	211	8.527	8.414	0.727	7.574	10.127
	Δln(GDP pc)			210	0.012	0.016	0.031	-0.114	0.091
	ln(Debt/GDP)			211	4.468	4.598	0.685	3.201	5.563
	ln(Debt/GDP)^2			211	20.432	21.139	6.057	10.247	30.946
	ln(Debt/GDP)^3			211	95.416	97.193	40.799	32.802	172.153
SWE	ln(GDP pc)	1800	2010	211	8.025	7.722	1.115	6.641	10.142
	Δln(GDP pc)			210	0.016	0.020	0.034	-0.094	0.120
	ln(Debt/GDP)			211	2.748	2.881	0.915	0.485	4.387
	ln(Debt/GDP)^2			211	8.385	8.297	4.605	0.235	19.246
	ln(Debt/GDP)^3			211	27.133	23.901	20.076	0.114	84.431
JPN	ln(GDP pc)	1872	2010	139	8.111	7.660	1.158	6.615	10.017
	Δln(GDP pc)			138	0.025	0.023	0.077	-0.681	0.162
	ln(Debt/GDP)			139	3.528	3.628	0.870	1.519	5.318
	ln(Debt/GDP)^2			139	13.197	13.164	6.066	2.307	28.280
	ln(Debt/GDP)^3			139	51.737	47.761	34.045	3.504	150.390

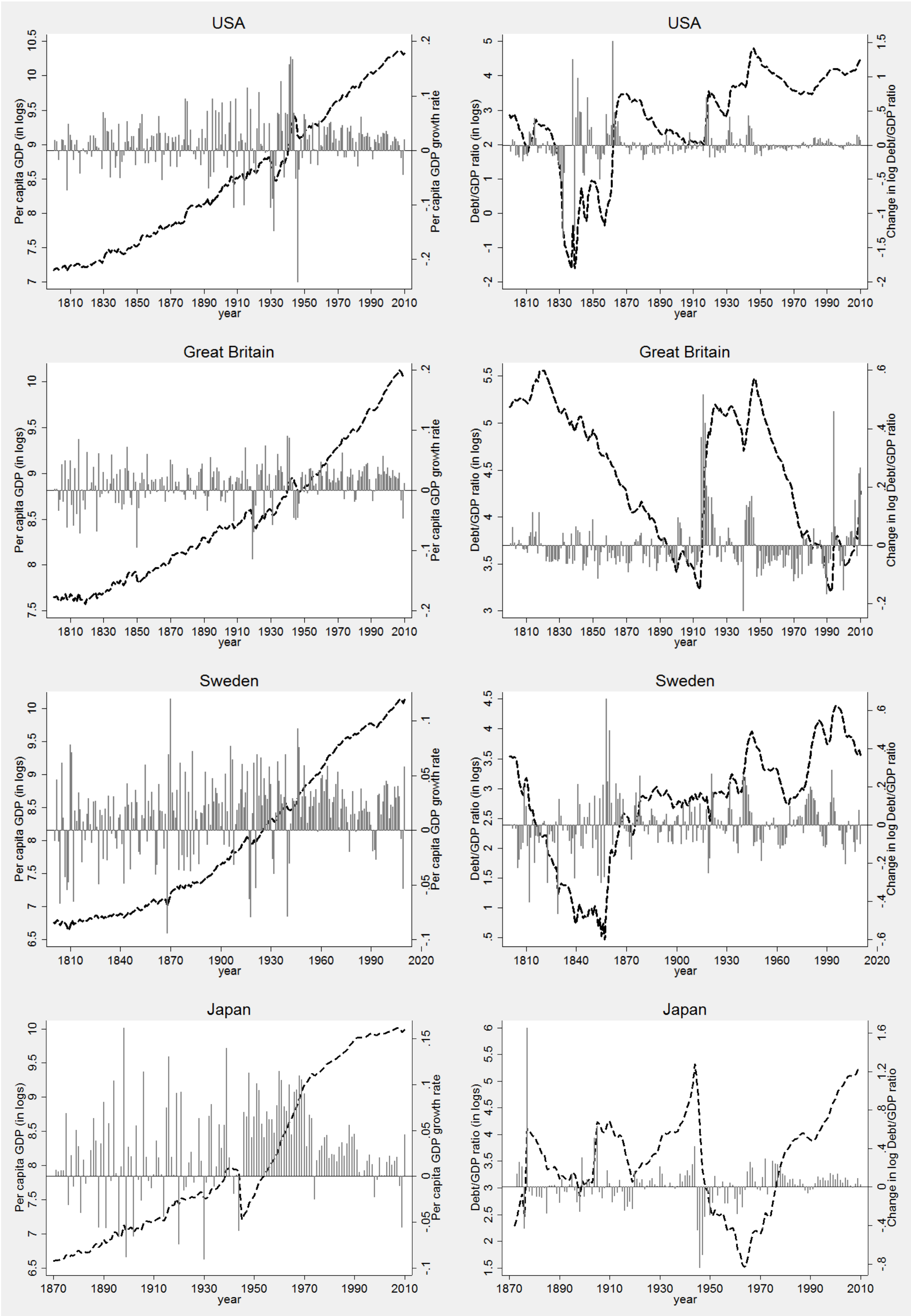
Notes: I provide the descriptive statistics for the levels variables included in our analysis (all in logarithms as indicated). Gap reports the number of missing observations. Δ is the difference operator.

Table A-2: Unit root/stationarity tests

		Start	End	Lags	DF-GLS	PP Z(t)	KPSS(i)	KPSS(ii)
USA	ln(GDP pc)	1800	2010	9	-0.88	0.65	0.47 ***	2.27 ***
	(Debt/GDP)			13	-2.55	-1.03	0.11	1.66 ***
	ln(Debt/GDP)			6	-2.38	-2.27	0.08	1.06 ***
	Δln(GDP pc)	1801	2010	8	-6.17***	-11.29***	0.03	0.14
	Δln(Debt/GDP)			5	-6.31***	-10.54***	0.03	0.04
GBR	ln(GDP pc)	1800	2010	6	0.09	2.26	0.66***	3.07***
	(Debt/GDP)			6	-1.89	-1.64	0.34***	0.94***
	ln(Debt/GDP)			1	-2.12	-1.66	0.93***	2.64***
	Δln(GDP pc)	1801	2010	8	-6.41***	-12.70***	0.03	0.52***
	Δln(Debt/GDP)			14	-2.08	-7.00***	0.08	0.10
SWE	ln(GDP pc)	1800	2010	10	-0.48	2.71	0.55***	1.99***
	(Debt/GDP)			10	-2.16	-1.95	0.12*	1.37***
	ln(Debt/GDP)			10	-1.66	-1.34	0.10	1.44***
	Δln(GDP pc)	1801	2010	9	-3.29**	-13.59***	0.14*	0.83***
	Δln(Debt/GDP)			9	-3.35**	-11.25***	0.10	0.12
JPN	ln(GDP pc)	1872	2010	0	-1.25	0.18	2.34***	13.30***
	(Debt/GDP)			2	-1.48	-0.88	0.39***	0.90***
	ln(Debt/GDP)			3	-1.76	-1.72	0.33***	0.39*
	Δln(GDP pc)	1873	2010	0	-10.77***	-10.76***	0.11	0.21
	Δln(Debt/GDP)			2	-4.54***	-9.78***	0.11	0.12

Notes: I use the Elliott, Rothenberg and Stock (1996) DF-GLS, Phillips and Perron (1988) PP and Kwiatkowski et al. (1992) KPSS tests. Lag length based on Ng and Perron (1995) criterion (DF-GLS and KPSS). KPSS(i) and (ii) have the null of trend and levels stationarity, respectively; DF-GLS and PP have the null of nonstationarity. Δ is the difference operator.

Figure A-1: Income and debt evolution



Notes: These plots chart the evolution of per capita GDP (in logs, left column) and the debt/GDP ratio (in logs, right column) for our four OECD countries. In each plot the levels variable (left axis, dashed line) is graphed alongside the variable in first differences (right axis, gray bars). Note that the per capita GDP growth series for Japan (-68%) excludes 1945 for ease of illustration.

Table A-3: Country Coverage (Extended Analysis)

Country	Argentina	Australia	Austria♯	Belgium♯	Brazil	Canada
Isocode	ARG	AUS	AUT	BEL	BRA	CAN
Start	1875	1861	1880	1846	1889	1870
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	20	12	0	0
Obs	136	150	111	153	122	141

Country	Chile	Colombia	Denmark	France	Germany‡	Greece♯
Isocode	CHL	COL	DNK	FRA	DEU	GRC
Start	1870	1899	1880	1880	1880	1848
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	23	37	15
Obs	141	112	131	108	94	163

Country	India	Italy	**Japan**	Netherlands	New Zealand♯	Norway
Isocode	IND	ITA	JPN	NLD	NZL	NOR
Start	1884	1861	1872	1815	1870	1880
End	2010	2010	2010	2010	2010	2010
Gaps	0	0	0	6	0	6
Obs	127	150	139	190	151	131

Country	Peru♯	Portugal♯	Spain♯	Sri Lanka	**Sweden**	Switzerland♯
Isocode	PER	PRT	ESP	LKA	SWE	CHE
Start	1883	1865	1850	1870	1800	1880
End	2010	2010	2010	2009	2010	2010
Gaps	14	0	4	35	0	16
Obs	114	146	157	105	211	115

Country	**Great Britain**	**United States**	Uruguay♯
Isocode	GBR	USA	URY
Start	1800	1800	1871
End	2010	2010	2009
Gaps	0	0	23
Obs	211	211	116

Notes: I present start and end years of per capita GDP and debt-ratio time series for the set of 27 countries for which I report the summability and co-summability results below. The countries in bold are studied in the main section of the paper. Excel files available at my personal website in due course will provide information on the country-time coverage by model.

A.3 Additional Empirical Results

I investigate the time series properties in a more diverse set of 23 additional economies (including some developing countries) in order to establish whether the patterns of results are fairly consistent across all countries investigated. The patterns observed in the summability analysis presented above are confirmed by results for the larger set of countries in Table A-4: in 23 out of 27 countries there is a pattern whereby we cannot reject the null that the per capita GDP *growth rate* is S(0) but reject this null in the equivalent *levels* series.[17] In 25 out of 27 countries all three debt variables reject summability of order zero.[18] Investigation of the co-summability results for the larger set of countries in Table A-5 again confirms that the patterns of results in the four OECD countries are qualitatively identical to those in the additional 23 countries investigated – only a single case (polynomial specification with linear, squared and cubed debt terms for Uruguay) satisfies co-summability. These results provide strong evidence against any nonlinear – or, for that matter, linear – long-run equilibrium relationship in all countries investigated.

In cointegration analysis the choice of the dependent variable has crucial bearings on the empirical results. I therefore also consider whether the same might be the case for my investigation of a nonlinear long-run relationship between debt and growth: I employ the debt-to-GDP ratio (in logs) as the dependent variable for models including the (log of) per capita GDP, and its squared and cubed polynomial terms as regressors in my analysis of co-summability. Adopting the debt-to-GDP ratio as dependent variable in balance and co-summability analysis in combination with polynomials of per capita GDP (results in Table A-6) finds only a single model – the nonlinear model with squared and cubed per capita income for Japan – co-summable. This confirms that my findings above are not the outcome of the (arbitrary) choice of the debt-to-GDP ratio as dependent variable.

The focus on empirical models limited to measures of income and debt may be overly simplistic and subject to serious misspecification. This aside these simply models do not have any solid justification from an economic theory point of view. In a further set of robustness checks I therefore

[17] For URY both series cannot reject S(0), for BRA, COL and PRT both series reject the S(0) null – see Table A-3 for country codes.

[18] For BRA all three polynomials cannot reject S(0), while (marginally) the same holds for the linear debt-to-GDP series for CAN.

extend the empirical model to include a number of determinants of growth as favoured by the cross-country growth literature. The following specifications are tested (obviously debt terms are always included): (i) I add inflation to the model, motivated by theoretical considerations of an investment-enhancing 'Tobin effect' of inflation as well as its reverse, and the strong negative impact found in cross-country empirical work (e.g. Barro, 1991); (ii) I analyse a 'Solow Model with Debt,' which includes the investment-to-GDP ratio (in logs) as well as the population growth rate and is motivated by the empirical equilibrium analysis of a standard Solow growth model in Mankiw, Romer and Weil (1992, equation (7) and Table I); (iii) I extend this to the 'Augmented Solow Model with Debt' by adding a measure of human capital (schooling) which was shown by the same authors to reconcile empirical estimates with observed income shares of capital and labour (equation (12) and Table II of that paper).

Summability test results for the new variables included are provided in Table A-7. Results here are not necessarily consistent across countries, in particular with regard to the population variable: the slow pace of demographic transition in advanced economies such as those studied here typically translates into the time series of this variable (in logarithms) appearing to be integrated of order two, I(2), with the growth rate thus I(1). This is the case for Sweden and Great Britain, where S(0) and thus I(0) is rejected, however the population growth rates for the United States and Japan cannot reject this property. There are also minor disagreements across countries with regards to the inflation and investment share (in logs). Table A-8 reports co-summability results from a model where inflation is added as additional regressor to the debt term(s). This setup is found to be co-summable across all models for Great Britain but not in any other country bar the threshold specification for the United States. The 'Solow Model with Debt' then finds all models for Sweden and Japan co-summable but none for the United States (again with the same exception of the 50% debt/GDP threshold) and Great Britain – see Table A-9. Finally, the 'Augmented Solow Model with Debt' in Table A-10 again finds all models for Sweden co-summable, yet none for the other countries, with the exception of the model with linear and squared debt terms for the United States and the two polynomial specifications for Great Britain.

Table A-4: Estimated Order of Summability – 27 countries

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
ARG	1875	2010	-	136	ln(GDP pc)	0.279	0.851	1.423
					∆ln(GDP pc)	-0.855	-0.249	0.357
					ln(Debt/GDP)	0.117	0.661	1.205
					ln(Debt/GDP) squared	0.075	0.691	1.308
					ln(Debt/GDP) cubed	0.117	0.727	1.337
AUS	1861	2010	-	150	ln(GDP pc)	0.262	0.847	1.432
					∆ln(GDP pc)	-0.222	0.464	1.151
					ln(Debt/GDP)	0.402	0.981	1.560
					ln(Debt/GDP) squared	0.300	0.978	1.656
					ln(Debt/GDP) cubed	0.308	1.011	1.714
AUT	1880	2010	20	111	ln(GDP pc)	0.056	0.775	1.495
					∆ln(GDP pc)	-0.552	0.104	0.760
					ln(Debt/GDP)	0.546	1.225	1.904
					ln(Debt/GDP) squared	0.473	1.190	1.907
					ln(Debt/GDP) cubed	0.476	1.158	1.839
BEL	1846	2010	12	153	ln(GDP pc)	0.347	0.730	1.113
					∆ln(GDP pc)	-0.484	0.163	0.810
					ln(Debt/GDP)	0.147	0.680	1.213
					ln(Debt/GDP) squared	0.131	0.675	1.220
					ln(Debt/GDP) cubed	0.106	0.673	1.240
BRA	1889	2010	-	122	ln(GDP pc)	0.650	1.157	1.664
					∆ln(GDP pc)	0.094	1.064	2.035
					ln(Debt/GDP)	-0.385	0.376	1.137
					ln(Debt/GDP) squared	-0.510	0.321	1.152
					ln(Debt/GDP) cubed	-0.596	0.244	1.084
CAN	1870	2010	-	141	ln(GDP pc)	0.101	0.552	1.003
					∆ln(GDP pc)	-1.235	-0.354	0.527
					ln(Debt/GDP)	-0.005	0.627	1.258
					ln(Debt/GDP) squared	0.024	0.657	1.291
					ln(Debt/GDP) cubed	0.136	0.690	1.244
CHL	1870	2010	-	141	ln(GDP pc)	0.145	0.717	1.289
					∆ln(GDP pc)	-0.584	0.022	0.628
					ln(Debt/GDP)	0.135	0.818	1.500
					ln(Debt/GDP) squared	0.176	0.828	1.480
					ln(Debt/GDP) cubed	0.216	0.859	1.501
COL	1899	2010	-	112	ln(GDP pc)	0.817	1.537	2.257
					∆ln(GDP pc)	0.374	1.157	1.940
					ln(Debt/GDP)	0.673	1.312	1.950
					ln(Debt/GDP) squared	0.745	1.243	1.740
					ln(Debt/GDP) cubed	0.766	1.179	1.593
DNK	1880	2010	-	131	ln(GDP pc)	0.141	0.677	1.213
					∆ln(GDP pc)	-0.573	0.155	0.882
					ln(Debt/GDP)	0.389	1.182	1.976
					ln(Debt/GDP) squared	0.484	1.228	1.972
					ln(Debt/GDP) cubed	0.514	1.240	1.967

Table continued on the following page

Table A-4: Estimated Order of Summability – 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
FRA	1880	2010	23	108	ln(GDP pc)	0.463	1.307	2.150
					Δln(GDP pc)	-0.490	0.226	0.942
					ln(Debt/GDP)	0.773	1.564	2.356
					ln(Debt/GDP) squared	1.153	2.336	3.519
					ln(Debt/GDP) cubed	1.557	3.156	4.755
DEU	1880	2010	37	94	ln(GDP pc)	0.549	1.239	1.929
					Δln(GDP pc)	-0.567	0.121	0.808
					ln(Debt/GDP)	0.514	0.940	1.367
					ln(Debt/GDP) squared	0.509	0.935	1.360
					ln(Debt/GDP) cubed	0.531	0.937	1.343
GRC	1848	2010	15	148	ln(GDP pc)	0.431	1.169	1.907
					Δln(GDP pc)	-0.534	0.138	0.809
					ln(Debt/GDP)	0.324	0.856	1.388
					ln(Debt/GDP) squared	0.406	0.960	1.515
					ln(Debt/GDP) cubed	0.367	0.990	1.613
IND	1884	2010	-	127	ln(GDP pc)	0.031	0.507	0.982
					Δln(GDP pc)	-0.925	-0.130	0.666
					ln(Debt/GDP)	0.279	0.991	1.702
					ln(Debt/GDP) squared	0.393	1.045	1.697
					ln(Debt/GDP) cubed	0.325	1.098	1.872
ITA	1861	2010	-	150	ln(GDP pc)	0.495	1.162	1.829
					Δln(GDP pc)	-0.210	0.364	0.939
					ln(Debt/GDP)	0.380	0.921	1.462
					ln(Debt/GDP) squared	0.384	0.937	1.491
					ln(Debt/GDP) cubed	0.390	0.953	1.517
JPN	1872	2010	-	139	ln(GDP pc)	0.987	2.390	3.792
					Δln(GDP pc)	-0.692	-0.004	0.683
					ln(Debt/GDP)	0.427	1.091	1.755
					ln(Debt/GDP) squared	0.433	1.101	1.769
					ln(Debt/GDP) cubed	0.410	1.114	1.819
NLD	1815	2010	6	190	ln(GDP pc)	0.055	0.569	1.083
					Δln(GDP pc)	-0.353	0.304	0.961
					ln(Debt/GDP)	0.462	1.084	1.705
					ln(Debt/GDP) squared	0.477	1.089	1.702
					ln(Debt/GDP) cubed	0.528	1.097	1.666
NZL	1870	2010	-	141	ln(GDP pc)	0.098	0.503	0.909
					Δln(GDP pc)	-0.323	0.299	0.920
					ln(Debt/GDP)	0.479	0.960	1.441
					ln(Debt/GDP) squared	0.419	0.986	1.553
					ln(Debt/GDP) cubed	0.486	1.009	1.533
NOR	1880	2010	6	125	ln(GDP pc)	0.656	1.349	2.042
					Δln(GDP pc)	-0.179	0.579	1.337
					ln(Debt/GDP)	0.398	1.073	1.749
					ln(Debt/GDP) squared	0.394	1.086	1.778
					ln(Debt/GDP) cubed	0.394	1.101	1.808

Table continued on the following page

Table A-4: Estimated Order of Summability – 27 countries (continued)

Country	Start Year	End Year	Gaps	Obs	Variable	CI low	$\hat{\delta}$	CI up
PER	1883	2010	14	114	ln(GDP pc)	0.284	0.820	1.357
					∆ln(GDP pc)	-0.073	0.665	1.404
					ln(Debt/GDP)	0.677	1.122	1.566
					ln(Debt/GDP) squared	0.683	1.063	1.444
					ln(Debt/GDP) cubed	0.676	1.009	1.342
PRT	1865	2010	-	146	ln(GDP pc)	0.464	1.087	1.709
					∆ln(GDP pc)	0.010	0.802	1.594
					ln(Debt/GDP)	0.397	0.933	1.470
					ln(Debt/GDP) squared	0.347	0.940	1.533
					ln(Debt/GDP) cubed	0.381	0.945	1.510
ESP	1850	2010	4	157	ln(GDP pc)	0.212	0.767	1.322
					∆ln(GDP pc)	-0.499	0.067	0.633
					ln(Debt/GDP)	0.394	0.994	1.595
					ln(Debt/GDP) squared	0.350	0.979	1.609
					ln(Debt/GDP) cubed	0.380	0.966	1.551
LKA	1870	2009	35	105	ln(GDP pc)	0.411	0.816	1.220
					∆ln(GDP pc)	-0.319	0.379	1.078
					ln(Debt/GDP)	0.210	0.771	1.332
					ln(Debt/GDP) squared	0.240	0.797	1.354
					ln(Debt/GDP) cubed	0.224	0.822	1.420
SWE	1800	2010	-	211	ln(GDP pc)	0.361	0.904	1.334
					∆ln(GDP pc)	-0.359	0.030	0.357
					ln(Debt/GDP)	0.637	1.624	2.603
					ln(Debt/GDP) squared	0.614	1.577	2.451
					ln(Debt/GDP) cubed	0.473	1.538	2.399
CHE	1880	2010	16	115	ln(GDP pc)	0.159	0.669	1.179
					∆ln(GDP pc)	-0.690	-0.097	0.497
					ln(Debt/GDP)	0.506	1.265	2.023
					ln(Debt/GDP) squared	0.508	1.254	2.001
					ln(Debt/GDP) cubed	0.477	1.255	2.033
GBR	1800	2010	-	211	ln(GDP pc)	0.731	1.696	2.662
					∆ln(GDP pc)	-0.444	0.126	0.695
					ln(Debt/GDP)	0.540	0.967	1.393
					ln(Debt/GDP) squared	0.509	0.948	1.386
					ln(Debt/GDP) cubed	0.475	0.931	1.387
USA	1800	2010	-	211	ln(GDP pc)	0.686	1.561	2.436
					∆ln(GDP pc)	-0.522	0.052	0.627
					ln(Debt/GDP)	0.551	1.082	1.613
					ln(Debt/GDP) squared	0.383	0.860	1.336
					ln(Debt/GDP) cubed	0.404	0.993	1.582
URY	1871	2009	23	116	ln(GDP pc)	-0.128	0.539	1.206
					∆ln(GDP pc)	-0.195	0.545	1.285
					ln(Debt/GDP)	0.419	1.061	1.704
					ln(Debt/GDP) squared	0.401	1.089	1.777
					ln(Debt/GDP) cubed	0.440	1.127	1.815

Notes: I report full sample order of summability estimates, CI low and up indicate the 95% confidence interval for the summability estimate $S(\hat{\delta})$ – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

Table A-5: Co-Summability – ln(GDP pc) specifications, 27 countries

						Co-Summability			
	Start	End	Gaps	obs	Nonlinearity	CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
ARG	1900	2008	-	109	-	0.543	0.990	1.438	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.343	0.928	1.512	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =99	ln(Debt/GDP)^3	0.423	0.936	1.449	$\hat{\delta}_{\hat{e}_t} \neq 0$
AUS	1861	2008	-	148	-	0.882	1.356	1.830	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.883	1.430	1.977	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =136	ln(Debt/GDP)^3	0.859	1.376	1.894	$\hat{\delta}_{\hat{e}_t} \neq 0$
AUT	1880	2008	2	109	-	0.444	1.012	1.580	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.218	0.760	1.301	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =99	ln(Debt/GDP)^3	0.101	0.579	1.057	$\hat{\delta}_{\hat{e}_t} \neq 0$
BEL	1846	2008	2	151	-	0.096	0.537	0.978	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.017	0.559	1.100	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =139	ln(Debt/GDP)^3	0.039	0.550	1.062	$\hat{\delta}_{\hat{e}_t} \neq 0$
BRA	1889	2008	-	120	-	1.022	1.832	2.643	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.687	1.298	1.908	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =109	ln(Debt/GDP)^3	0.684	1.056	1.428	$\hat{\delta}_{\hat{e}_t} \neq 0$
CAN	1870	2008	-	139	-	0.223	0.770	1.318	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.233	0.751	1.269	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =127	ln(Debt/GDP)^3	0.242	0.751	1.259	$\hat{\delta}_{\hat{e}_t} \neq 0$
CHL	1827	2008	-	182	-	0.870	1.424	1.978	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =14	ln(Debt/GDP)^2	0.871	1.418	1.965	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =169	ln(Debt/GDP)^3	0.877	1.488	2.099	$\hat{\delta}_{\hat{e}_t} \neq 0$
COL	1900	2008	-	109	-	0.810	1.265	1.720	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.424	0.946	1.467	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =99	ln(Debt/GDP)^3	0.054	0.639	1.224	$\hat{\delta}_{\hat{e}_t} \neq 0$
DNK	1880	2008	-	129	-	1.049	1.801	2.553	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.589	1.654	2.719	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =118	ln(Debt/GDP)^3	0.695	1.604	2.513	$\hat{\delta}_{\hat{e}_t} \neq 0$
FRA	1880	2008	2	106	-	0.393	1.073	1.753	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.329	1.108	1.887	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =96	ln(Debt/GDP)^3	0.392	1.040	1.689	$\hat{\delta}_{\hat{e}_t} \neq 0$
DEU	1880	2008	2	106	-	0.330	0.757	1.185	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.353	0.959	1.564	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =96	ln(Debt/GDP)^3	0.326	0.893	1.459	$\hat{\delta}_{\hat{e}_t} \neq 0$
IND	1884	2008	-	125	-	0.164	0.691	1.218	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.163	0.691	1.218	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =114	ln(Debt/GDP)^3	0.335	0.751	1.167	$\hat{\delta}_{\hat{e}_t} \neq 0$
ITA	1861	2008	-	148	-	0.704	1.124	1.544	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.473	0.975	1.477	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =136	ln(Debt/GDP)^3	0.526	1.058	1.591	$\hat{\delta}_{\hat{e}_t} \neq 0$
JPN	1872	2008	2	123	-	0.462	1.135	1.809	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.194	0.848	1.503	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =112	ln(Debt/GDP)^3	0.200	0.859	1.518	$\hat{\delta}_{\hat{e}_t} \neq 0$

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Table A-5: Co-Summability – ln(GDP pc) specifications, 27 countries (cont'd)

	Start	End	Gaps	obs	Nonlinearity	Co-Summability			
						CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
NLD	1820	2008	1	183	-	0.210	0.595	0.980	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =15	ln(Debt/GDP)^2	0.238	0.640	1.042	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =169	ln(Debt/GDP)^3	0.291	0.708	1.124	$\hat{\delta}_{\hat{e}_t} \neq 0$
NZL	1870	2008	-	139	-	0.587	0.872	1.158	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.545	0.832	1.119	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =127	ln(Debt/GDP)^3	0.470	0.809	1.149	$\hat{\delta}_{\hat{e}_t} \neq 0$
NOR	1880	2008	1	123	-	0.721	1.325	1.930	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.476	1.161	1.846	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =112	ln(Debt/GDP)^3	0.540	1.221	1.902	$\hat{\delta}_{\hat{e}_t} \neq 0$
PER	1896	2008	4	99	-	0.327	0.922	1.517	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.319	0.890	1.460	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =89	ln(Debt/GDP)^3	0.261	0.877	1.494	$\hat{\delta}_{\hat{e}_t} \neq 0$
PRT	1865	2008	-	144	-	0.304	0.790	1.275	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.423	0.782	1.140	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =132	ln(Debt/GDP)^3	0.132	0.615	1.098	$\hat{\delta}_{\hat{e}_t} \neq 0$
SWE	1820	2008	-	189	-	0.331	0.928	1.524	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =15	ln(Debt/GDP)^2	0.281	0.854	1.426	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =175	ln(Debt/GDP)^3	0.280	0.957	1.634	$\hat{\delta}_{\hat{e}_t} \neq 0$
CHE	1880	2008	3	113	-	0.699	1.173	1.647	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.673	1.173	1.673	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =102	ln(Debt/GDP)^3	0.548	1.054	1.560	$\hat{\delta}_{\hat{e}_t} \neq 0$
ESP	1850	2008	1	155	-	0.251	0.911	1.571	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.219	0.849	1.478	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =143	ln(Debt/GDP)^3	0.300	0.862	1.423	$\hat{\delta}_{\hat{e}_t} \neq 0$
LKA	1870	2008	1	104	-	0.248	0.702	1.157	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =11	ln(Debt/GDP)^2	0.196	0.740	1.285	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =94	ln(Debt/GDP)^3	0.157	0.700	1.243	$\hat{\delta}_{\hat{e}_t} \neq 0$
GBR	1830	2008	-	179	-	0.600	0.995	1.390	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =14	ln(Debt/GDP)^2	0.580	1.014	1.448	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =166	ln(Debt/GDP)^3	0.425	0.951	1.477	$\hat{\delta}_{\hat{e}_t} \neq 0$
USA	1870	2008	-	139	-	0.483	1.116	1.748	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =13	ln(Debt/GDP)^2	0.415	1.090	1.764	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =127	ln(Debt/GDP)^3	0.344	0.943	1.541	$\hat{\delta}_{\hat{e}_t} \neq 0$
URY	1871	2008	1	115	-	0.218	0.602	0.986	$\hat{\delta}_{\hat{e}_t} \neq 0$
				b =12	ln(Debt/GDP)^2	0.156	0.602	1.048	$\hat{\delta}_{\hat{e}_t} \neq 0$
				M =104	ln(Debt/GDP)^3	-0.119	0.477	1.074	$\hat{\delta}_{\hat{e}_t} = 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_{\hat{e}_t} \neq (=)0$ implies that co-summability is (not) rejected. $b = int\sqrt{T} + 1$ refers to the time series length of the subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).

Table A-6: Co-Summability – ln(debt) as dependent variable

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	0.340	0.820	1.300	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(GDP pc)^2	0.246	0.886	1.527	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(GDP pc)^3	0.284	0.885	1.485	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1800	2010	211	-	0.560	1.263	1.965	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(GDP pc)^2	0.501	1.173	1.846	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(GDP pc)^3	0.369	0.917	1.466	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	211	-	0.709	1.623	2.538	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(GDP pc)^2	0.601	1.534	2.467	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(GDP pc)^3	0.590	1.515	2.439	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	139	-	0.145	0.669	1.193	$S(\delta_{\hat{e}_t}) \neq 0$
			b =13	ln(GDP pc)^2	0.204	0.806	1.408	$S(\delta_{\hat{e}_t}) \neq 0$
			M =127	ln(GDP pc)^3	-0.160	0.338	0.837	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models I take the debt-to-GDP ratio (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_{\hat{e}_t} \neq (=)0$ implies that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(GDP pc)^2 also includes ln(GDP pc), while the model with ln(GDP pc)^3 also includes ln(GDP pc)^2 and ln(GDP pc).

Table A-7: Estimated Order of Summability – Additional Covariates

Country	Start & End Year		n	Variable	CI low	$\hat{\delta}$	CI up
USA	1800	2010	211	Inflation	-0.572	0.035	0.642
	1870	2010	141	Δln(Population)	-0.130	0.417	0.964
				ln(Invest/GDP)	0.520	1.407	2.294
	1880	2010	131	Schooling	1.234	2.471	3.708
GBR	1800	2010	211	Inflation	0.457	1.210	1.963
	1850	2010	161	Δln(Population)	0.103	0.590	1.078
				ln(Invest/GDP)	-0.178	0.386	0.950
	1870	2010	141	Schooling	0.918	1.696	2.474
SWE	1800	2010	211	Inflation	0.492	1.376	2.260
	1801	2010	210	Δln(Population)	0.131	0.644	1.158
				ln(Invest/GDP)	0.211	0.919	1.627
	1870	2010	141	Schooling	1.094	2.153	3.212
JPN	1800	2010	211	Inflation	0.030	0.608	1.185
	1850	2010	161	Δln(Population)	-0.269	0.540	1.350
				ln(Invest/GDP)	0.027	0.529	1.031
	1870	2010	141	Schooling	0.368	1.058	1.749

Notes: CI low and up indicate the 95% confidence interval for the summability estimate $S(\delta)$ constructed from subsampling – shaded cells indicate variable series where the summability confidence interval includes zero. In all tests conducted I allow for deterministic terms (constant and trend).

Table A-8: Co-Summability – Debt and Inflation Model

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\hat{\delta}}_{\hat{e}_t}$	CI up	Verdict
USA	1800	2010	211	-	0.417	0.951	1.486	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(Debt/GDP)^2	0.312	0.892	1.472	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(Debt/GDP)^3	0.353	0.855	1.357	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	-0.169	0.268	0.706	$S(\delta_{\hat{e}_t}) = 0$
GBR	1800	2010	211	-	-0.118	0.248	0.614	$S(\delta_{\hat{e}_t}) = 0$
			b =16	ln(Debt/GDP)^2	-0.146	0.246	0.639	$S(\delta_{\hat{e}_t}) = 0$
			M =196	ln(Debt/GDP)^3	-0.178	0.245	0.669	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-0.169	0.268	0.706	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 70%	-0.146	0.302	-0.751	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 90%	-0.107	0.304	0.715	$S(\delta_{\hat{e}_t}) = 0$
SWE	1800	2010	211	-	0.386	0.924	1.463	$S(\delta_{\hat{e}_t}) \neq 0$
			b =16	ln(Debt/GDP)^2	0.354	0.949	1.544	$S(\delta_{\hat{e}_t}) \neq 0$
			M =196	ln(Debt/GDP)^3	0.337	0.989	1.641	$S(\delta_{\hat{e}_t}) \neq 0$
JPN	1872	2010	125	-	0.770	1.589	2.409	$S(\delta_{\hat{e}_t}) \neq 0$
			b =12	ln(Debt/GDP)^2	0.228	0.783	1.338	$S(\delta_{\hat{e}_t}) \neq 0$
			M =114	ln(Debt/GDP)^3	0.109	0.739	1.369	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.388	1.029	1.670	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).

Table A-9: Balance and Co-Summability – Solow Model with Debt

	Start	End	Obs	Nonlinearity	Co-Summability CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1870	2010	141	-	0.212	0.888	1.565	$S(\delta_{\hat{e}_t}) \neq 0$
			b =13	ln(Debt/GDP)^2	0.158	0.745	1.332	$S(\delta_{\hat{e}_t}) \neq 0$
			M =129	ln(Debt/GDP)^3	0.125	0.939	1.753	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	-0.321	0.213	0.747	$S(\delta_{\hat{e}_t}) = 0$
GBR	1850	2010	161	-	0.155	1.003	1.851	$S(\delta_{\hat{e}_t}) \neq 0$
			b =14	ln(Debt/GDP)^2	0.148	0.820	1.492	$S(\delta_{\hat{e}_t}) \neq 0$
			M =148	ln(Debt/GDP)^3	0.117	0.754	1.392	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.209	0.884	1.559	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 70%	0.343	0.972	1.602	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 90%	0.345	1.059	1.773	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1800	2010	211	-	-0.152	0.334	0.820	$S(\delta_{\hat{e}_t}) = 0$
			b =16	ln(Debt/GDP)^2	-0.372	0.284	0.940	$S(\delta_{\hat{e}_t}) = 0$
			M =196	ln(Debt/GDP)^3	-0.540	0.195	0.931	$S(\delta_{\hat{e}_t}) = 0$
JPN	1872	2010	125	-	-0.253	0.246	0.745	$S(\delta_{\hat{e}_t}) = 0$
			b =12	ln(Debt/GDP)^2	-0.277	0.239	0.754	$S(\delta_{\hat{e}_t}) = 0$
			M =114	ln(Debt/GDP)^3	-0.262	0.232	0.726	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	-0.014	0.517	1.048	$S(\delta_{\hat{e}_t}) = 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the balance and co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_y \neq (=)\hat{\delta}_z$ implies that balance is (not) rejected, $\hat{\delta}_{\hat{e}_t} \neq (=)0$ that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).

Table A-10: Balance and Co-Summability – Augmented Solow Model with Debt

	Start	End	Obs	Nonlinearity	Co-Summability			
					CI low	$\hat{\delta}_{\hat{e}_t}$	CI up	Verdict
USA	1880	2010	131	-	0.304	0.892	1.479	$S(\delta_{\hat{e}_t}) \neq 0$
			b =12	ln(Debt/GDP)^2	-0.011	0.619	1.250	$S(\delta_{\hat{e}_t}) = 0$
			M =120	ln(Debt/GDP)^3	0.059	0.663	1.267	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.144	0.720	1.295	$S(\delta_{\hat{e}_t}) \neq 0$
GBR	1870	2010	141	-	0.275	0.915	1.554	$S(\delta_{\hat{e}_t}) \neq 0$
			b =13	ln(Debt/GDP)^2	-0.183	0.368	0.919	$S(\delta_{\hat{e}_t}) = 0$
			M =129	ln(Debt/GDP)^3	-0.010	0.634	1.278	$S(\delta_{\hat{e}_t}) = 0$
				Threshold 50%	0.230	0.853	1.475	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 70%	0.343	0.972	1.602	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 90%	0.345	1.059	1.773	$S(\delta_{\hat{e}_t}) \neq 0$
SWE	1870	2010	141	-	-0.078	0.437	0.952	$S(\delta_{\hat{e}_t}) = 0$
			b =13	ln(Debt/GDP)^2	-0.233	0.387	1.007	$S(\delta_{\hat{e}_t}) = 0$
			M =129	ln(Debt/GDP)^3	-0.185	0.340	0.866	$S(\delta_{\hat{e}_t}) = 0$
JPN	1890	2010	107	-	0.055	0.533	1.011	$S(\delta_{\hat{e}_t}) \neq 0$
			b =11	ln(Debt/GDP)^2	0.069	0.509	0.950	$S(\delta_{\hat{e}_t}) \neq 0$
			M =97	ln(Debt/GDP)^3	0.065	0.509	0.952	$S(\delta_{\hat{e}_t}) \neq 0$
				Threshold 50%	0.435	0.790	1.145	$S(\delta_{\hat{e}_t}) \neq 0$

Notes: In all models I take per capita GDP (in logarithms) as the dependent variable. CI low and up indicate the 95% confidence interval for the co-summability estimates. In all tests conducted I allow for deterministic terms (constant and trend). $\hat{\delta}_{\hat{e}_t} \neq (=)0$ implies that co-summability is (not) rejected. Obs reports the number of observations, $b = int\sqrt{T} + 1$ refers to the time series length of each subsample, $M = T - b + 1$ to the number of subsamples used in the analysis. Regarding the 'Nonlinearity,' the model with ln(Debt/GDP)^2 also includes ln(Debt/GDP), while the model with ln(Debt/GDP)^3 also includes ln(Debt/GDP)^2 and ln(Debt/GDP).