# Accounting for unobserved heterogeneity in panel time series models\*

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**Abstract:** This note introduces the Augmented Mean Group (AMG) estimator for the analysis of macro panel data in the presence of slope heterogeneity, variable nonstationarity and cross-section dependence. Building on extensive Monte Carlo simulations we show that its performance matches that of the popular Pesaran (2006) Common Correlated Effects (CCE) estimators in a wide range of setups. An empirical application highlights the merits of the AMG approach for cross-country productivity analysis.

**Keywords:** Nonstationary Panel Econometrics, Common Factor Models, Cross-Section Dependence

**JEL codes:** C23, O47

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## **1** Introduction

In this note we consider the estimation of mean slope coefficients in a linear heterogeneous panel model where unobserved common factors lead to correlations in the disturbances across units and to correlations between the disturbances and the regressors. We introduce the Augmented Mean Group (AMG) estimator and employ Monte Carlo experiments to investigate its small sample performance as well as that of a number of pooled and heterogeneous parameter estimators — including the popular Pesaran (2006) common correlated effects (CCE) estimators — in various contexts, including that of variable and factor nonstationarity.

Our investigation is motivated by the range of parameter estimates obtained from cross-country production function regressions for the manufacturing and agriculture sectors (see Eberhardt, Helmers, & Strauss, 2013; Eberhardt & Teal, 2012, 2013, 2014), where the combination of technology heterogeneity, variable nonstationarity and cross-section dependence (CSD) can lead to severe distortions in the estimates from standard panel estimators. In the present note we provide further evidence as to the importance of accounting for heterogeneity, nonstationarity and cross-section dependence in macro panel data, taking the viewpoint of the applied econometrician. Building on Coakley, Fuertes, and Smith (2006) and Kapetanios, Pesaran, and Yamagata (2011), we highlight the practical shortcomings of the Monte Carlo Data Generating Processes (DGPs) adopted in these studies. Our own simulations provide patterns of results more in line with those observed in the applied cross-country empirical literature. A brief empirical application highlights the merits of the AMG approach.

The remainder of this note is organised as follows: Section 2 introduces the empirical framework and the Augmented Mean Group (AMG) estimator. In Section 3 we discuss our simulation setup, four scenarios for analysis as well as the Monte Carlo results. An empirical illustration is provided in Section 4. The final section concludes.

### 2 The Augmented Mean Group estimator

We adopt the following empirical model: for i = 1, ..., N, t = 1, ..., T and m = 1, ..., k,

$$y_{it} = \boldsymbol{\beta}'_{i} \boldsymbol{x}_{it} + u_{it} \qquad u_{it} = \alpha_{i} + \boldsymbol{\lambda}'_{i} \boldsymbol{f}_{t} + \varepsilon_{it}$$
(1)

$$x_{mit} = \pi_{mi} + \boldsymbol{\delta}'_{mi} \boldsymbol{g}_{mt} + \rho_{1mi} f_{1mt} + \ldots + \rho_{nmi} f_{nmt} + v_{mit}$$
(2)

where 
$$f_{\cdot mt} \subset f_t$$
  $f_t = \varrho' f_{t-1} + \epsilon_t$  and  $g_t = \kappa' g_{t-1} + \epsilon_t$  (3)

where  $x_{it}$  is a vector of observable covariates. The unobservables  $u_{it}$  are modelled as a combination of group-specific effects  $\alpha_i$  and a set of common factors  $f_t$  with group-specific factor loadings  $\lambda_i$ . In equation (2) we provide a representation of the k observable regressors, which

are modelled as linear functions of unobserved common factors  $f_t$  and  $g_t$ , with respective group-specific factor loadings. Equation (3) specifies the evolution of the unobserved factors. We maintain the following assumptions for the general model and the data it is applied to:

- A.1 The  $\beta_i$  parameters are unknown random coefficients with fixed means and finite variances:  $\beta_i = \beta + \eta_i$  where  $\eta_i \sim iid(0, \Omega_\eta)$ . Similarly for the factor loadings.<sup>1</sup>
- A.2 Error terms  $\varepsilon_{it} \sim N(0, \sigma^2)$ , where  $\sigma^2$  is finite. Similarly for  $v_{mit}$  and  $\epsilon_t$ .
- A.3 Observables  $x_{it}$  and  $y_{it}$ , as well as the unobserved common factors  $f_t$  and  $g_t$  are not *a* priori assumed to be stationary:  $|\boldsymbol{\varrho}| \leq 1$ ,  $|\boldsymbol{\kappa}| \leq 1$ .
- A.4 The unobserved common factors with heterogeneous factor loadings  $\lambda'_i f_t$ , can contain elements which are common across groups as well as elements which are group-specific.
- A.5 There is an overlap between the unobserved common factors driving y and x ( $f_{\cdot mt} \subset f_t$ ), creating serious difficulties for the identification of  $\beta_i$  or its mean.

The most important features of this setup are (i) the potential nonstationarity of observables and unobservables  $(y_{it}, x_{it}, f_t, g_t)$ , and (ii) the potential heterogeneity in the impact of observables and unobservables across panel groups  $(\alpha_i, \beta_i, \lambda_i)$ . Taken together these properties have important bearings on estimation and inference in macro panel data (Coakley et al., 2006; Pesaran, 2006; Kapetanios et al., 2011; Chudik & Pesaran, 2013).

We now introduce a novel estimation approach, the Augmented Mean Group (AMG) estimator, which accounts for cross-section dependence by inclusion of a 'common dynamic effect' in the group-specific regressions. This variable is constructed from the period dummy coefficients of a pooled regression in first differences and represents the levels-equivalent average evolution of unobserved common factors across all panel groups. Provided the unobserved common factors form part of the group-specific cointegrating relation — this matches the assumption of the Pesaran (2006) CCE estimators (Pedroni, 2007) — the augmented regression model captures this relationship.

AMG Stage (i) 
$$\Delta y_{it} = \boldsymbol{b'} \Delta \boldsymbol{x}_{it} + \sum_{t=2}^{T} c_t \Delta D_t + e_{it} \Rightarrow \hat{\boldsymbol{c}}_t \equiv \hat{\mu}_t^{\bullet}$$
 (4)

AMG Stage (ii) 
$$y_{it} = a_i + \boldsymbol{b}'_i \boldsymbol{x}_{it} + c_i t + d_i \hat{\mu}^{\bullet}_t + e_{it} \Rightarrow \hat{\boldsymbol{b}}_{AMG} = N^{-1} \sum_i \hat{\boldsymbol{b}}_i$$
 (5)

The first stage represents a standard Ordinary Least Squares (OLS) regression in first difference (FD-OLS) with T - 1 period dummies in first differences  $\Delta D_t$ , from which we collect the estimated coefficients which are relabelled as  $\hat{\mu}_t^{\bullet}$ . This process is extracted from the pooled regression *in first differences* since nonstationary variables and unobservables can severely bias

<sup>&</sup>lt;sup>1</sup>The assumption of random coefficients is for convenience. Based on the findings by Pesaran and Smith (1995, footnote 2, p.81) the coefficients could alternatively be fixed but differing across groups.

the estimates in a pooled levels regressions. In the second stage this constructed variable  $\hat{\mu}_t^{\bullet}$  is included in each of the N group-specific regressions, which also include linear trend terms to capture omitted idiosyncratic processes which evolve in a linear fashion over time. Alternatively (not shown) we can subtract  $\hat{\mu}_t^{\bullet}$  from the dependent variable, which implies the common process is imposed on each group with unit coefficient — AMG(i) below.<sup>2</sup> In either case the AMG estimates are derived as averages of the individual  $\hat{\beta}_i$  estimates, following the Pesaran and Smith (1995) Mean Group approach.

The AMG is closely related to the approach in Coakley, Fuertes, and Smith (2002), which included principal components extracted from pooled OLS regression residuals in second-stage group-specific regressions following the standard Pesaran and Smith (1995) Mean Group model.<sup>3</sup> It is also related to the Mean Group version of the Pesaran (2006) CCE estimators (CMG), which augments the group-specific regression with panel cross-section averages of the dependent and independent variables. The AMG approach uses an explicit rather than implicit estimate for  $f_t$  from the pooled first stage regression: from (1) we obtain

$$\Delta y_{it} = \beta_i \Delta \boldsymbol{x}_{it} + \boldsymbol{\lambda}'_i \Delta \boldsymbol{f}_t + \Delta u_{it} \tag{6}$$

Pooled estimation of this model yields the common or mean evolution of unobservables in levels across panel groups over time  $\hat{\mu}_t^{\bullet}$ . Our pooled estimate is some function  $h(\cdot)$  of the unobserved common factors  $f_t$ :  $\hat{\mu}_t^{\bullet} = h(\bar{\lambda} f_t)$ . Plugging this estimate back into the model in equation (1) yields

$$y_{it} = \alpha_i + \beta_i \boldsymbol{x}_{it} + \boldsymbol{\lambda}_i h(\boldsymbol{\lambda} \boldsymbol{f}_t) + u_{it}$$
(7)

From equation (7) we can see that provided there are no issues related to  $\Delta u_{it}$  in the first stage regression — see simulation setup (iii) below — the estimate  $\hat{\mu}_t^{\bullet}$  obtained can be included in the second stage to account for the unobserved common factors  $f_t$  and allow for their heterogeneous impact on  $y_{it}$ . The AMG and CMG share a simplicity of implementation — both represent standard MG models augmented with additional covariates and estimated by OLS — which makes them particularly attractive for applied work.

### **3** Simulations

We conducted initial Monte Carlo simulations following the setups in Coakley et al. (2006) and Kapetanios et al. (2011), finding that the use of 'naïve estimators' ignoring slope heterogeneity

<sup>&</sup>lt;sup>2</sup>Note that unity is also the Mean Group average estimate expected for the coefficients on  $\hat{\mu}_t^{\bullet}$ :  $\bar{d} = N^{-1} \sum_i d_i = 1$ . The period dummy coefficients which make up  $\hat{\mu}_t^{\bullet}$  represent an average of the unobservables and their group-specific factor loadings across groups.

<sup>&</sup>lt;sup>3</sup>Coakley et al. (2002) laid the foundation for more recent implementations which are based on the same intuition but carry out iterations of the two steps to avoid the serious bias introduced in the misspecified pooled OLS regression (Bai & Kao, 2006; Bai, Kao, & Ng, 2009).

and the factor setup (pooled OLS or 2-way fixed effects estimators) did not create considerable bias *provided* their empirical implementation included period dummies — a standard practice in the applied literature. This is surprising, since these estimators impose common factor loadings across groups on all unobserved common factors. This analysis (setups and results are available in an Online Appendix) would suggest that estimators which account for cross-section dependence do not yield a dramatically different result in terms of bias from standard pooled estimators which ignore this dependence.<sup>4</sup> This conclusion is at odds with the experience in applied work, where we see vast differences in coefficient estimates between the pooled OLS and Fixed Effects estimates on the one hand, and the CCE and AMG-type estimators on the other — see our empirical illustration in Section 4. In the following we therefore consider a number of alternative scenarios which create patterns of results somewhat more in line with those observed in real data applications.

We define our dependent variable and regressor as

$$y_{it} = \beta_i x_{it} + u_{it} \qquad u_{it} = \alpha_i + \lambda_{i1}^y f_{1t} + \lambda_{i2}^y f_{2t} + \varepsilon_{it}$$
(8)

$$x_{it} = a_i + \lambda_{i1}^x f_{1t} + \lambda_{i3}^x f_{3t} + \epsilon_{it} \qquad \epsilon_{it} = \rho \epsilon_{i,t-1} + e_{it}$$
(9)

The serially-correlated x-variable is in practice constructed using a dynamic equation

$$x_{it} = (1 - \rho)a_i + \lambda_{i1}^x f_{1t} - \rho \lambda_{i1}^x f_{1,t-1} + \lambda_{i3}^x f_{3t} - \rho \lambda_{i3}^x f_{3,t-1} + \rho x_{i,t-1} + e_{it}$$

which we begin with  $x_{i,-49} = a_i$  and then accumulate for  $t = -48, \ldots, 0, 1, \ldots, T$ , discarding the first 50 time-series observations for all *i*. The common AR-coefficient is  $\rho = .25$ .

The unobserved common factors are nonstationary processes with individual drifts so as to ensure upward evolution over time, as observed in many macro data series.

$$f_{jt} = \mu_j + f_{j,t-1} + \upsilon_{fjt} \qquad t = -48, \dots, 0, 1, \dots, T \qquad f_{j,-49} = 0$$
(10)  
$$\upsilon_{fjt} \sim N\left(0, \sigma_{fj}^2\right) \qquad \sigma_{fj}^2 = .00125 \qquad \mu_j = \{0.015, 0.012, 0.01\} \qquad j = 1, 2, 3$$

The error terms for the y and x equations are defined as

$$\begin{array}{ll} e_{it} & \sim & iid \, N(0, \sigma_{e,i}^2) & \text{ where } \sigma_{e,i}^2 \sim U[.001, .003] \\ \varepsilon_{it} & \sim & iid \, N(0, \sigma_{\varepsilon}^2) & \sigma_{\varepsilon}^2 = .00125 \end{array}$$

The slope coefficient on x is set to  $\beta_i = 1 + e_i^{\beta}$  where  $e_i^{\beta} \sim U[-.25, +.25]$ . The factor loadings are uniformly distributed, with  $\lambda_{i1}^x$  and  $\lambda_{i1}^y$  iid U[0, 1] respectively, and  $\lambda_{i3}^x$  and  $\lambda_{i2}^y$  iid U[.25, 1.25] respectively.

<sup>&</sup>lt;sup>4</sup>Coakley et al. (2006) were the first to point out the surprising limited bias for the two-way fixed effects estimator in their simulations.

We consider the following cases:

- (i) baseline (as above),
- (ii) baseline with additional group-specific linear trends,
- (iii) feedbacks: an idiosyncratic shock to y feeds back into x with one period lag, and
- (iv) two 'clubs' of countries with the same  $\beta$  coefficient.

The group-specific linear trends in Case (ii) are distributed U[-.02, +.03], s.t. that the mean annual growth rate across the panel is non-zero. For the feedback case, the lagged error  $\varepsilon_{i,t-1}$ from the *y*-equation in (8) is included in the *x*-equation in (9) with coefficient .25 (in practice we enter this term in the same way as the other terms in the dynamic equation as described above). Finally, for the 'two clubs' case 20% of panel groups have  $\beta = 2$ , while 80% have  $\beta = .75$ , s.t. the mean  $\beta$  across all groups is still unity.

#### — Table 1 about here —

Results for the case of T = 30 and N = 50 are presented in Table 1. This combination was chosen to match the dimensions of the panel in the empirical illustration (Section 4). Results for other combinations of N and T can be found in an Online Appendix.

Results for our benchmark specification — Case (i) — indicate that 2FE has bias of .0324 with empirical standard error of .0876, compared to .0271 for the infeasible FE estimator. Similarly for the POLS and MG estimator. In all cases this bias is increasing in T and decreasing in N. For the CCE and AMG estimators, all unbiased, the AMG(ii) is commonly most efficient.

Once we add the idiosyncratic trend terms — Case (ii) — the bias in the standard pooled estimators does not change by any significant margin. 2FE now has a bias of .0277, but a very substantial empirical standard error of .1973 (more than double that of the benchmark case), compared with .0280 for the infeasible FE estimator. This imprecision increases with T. In contrast the unbiased CCE and AMG estimators are still efficient.

By construction, the feedback setup — Case (iii) — leads to bias in the FD-OLS, which carries over to the AMG estimators: due to differencing the  $\varepsilon_{i,t-1}$  are contained in both the errors and the regressors of the FD-OLS estimation equation, whereas this is not the case in the other (levels-based) estimators which account for common factors. We therefore also present the results for an IV-version of the FD-OLS estimator, where we use growth rates at time t - 1as instruments for the endogenous growth rates at time t (FD-IV), and AMG estimators which are based on the year dummies from this instrumented first stage regression (AMG-IV). The pooled OLS, 2FE and MG results are virtually unchanged from the baseline results: 2FE has a bias of .0299 with empirical standard error of .0865 compared with .0271 for the infeasible FE estimator. The augmented estimators all display small finite sample bias, albeit very modest in case of the CCE estimators, while the new AMG estimates based on the FD-IV results are unbiased. The latter is unbiased, but inefficient compared with the new AMG estimators. In the setup where  $\beta$  is heterogeneous but only takes two values for different 'clubs' of panel groups — Case (iv) — the results show considerable bias for the POLS estimator, while other estimators remain relatively unchanged: the 2FE estimator has a bias of .0224 and an empirical standard error of .1375 compared with .0357 for the infeasible FE. The small finite sample bias for the AMG(ii) implementation is wiped out in the instrumented version AMG(ii)-IV.

In addition to these cases we also experimented with (a) normally distributed  $\beta_i$  and factor loadings, (b) large variation in factor loadings, (c) large variation in slope coefficients, as well as various combinations of these cases. We also analysed a version of the AMG where we allowed for slope heterogeneity in the first stage estimation via interaction terms. All of these results (contained in an Online Appendix) confirm the performance of the AMG estimators while highlighting more substantial bias in the naïve estimators (POLS, 2FE, MG).

## 4 Empirical Illustration

In Table 2 we present results from Cobb-Douglas production function regressions in a sample of 48 developing and developed countries for 1970–2002. The data are taken from UNIDO (2004) and described in more detail in Eberhardt and Teal (2012). Of the pooled estimators in Panel (A) only the FD-OLS results in stationary and cross-sectionally independent residuals. Its capital coefficient, at around .3, is in line with income share data (Gollin, 2002). Note the inflated capital coefficients in the POLS and 2FE models where residual diagnostics indicate serious misspecification. Among the heterogeneous estimators the MG and standard CMG display cross-sectionally dependent residuals, while the AMG and CMG with additional id-iosyncratic trend terms yield favourable diagnostics and mean capital coefficients close to .3. Closer investigation of these estimates indicates that they are not driven by outliers.

#### — Table 2 and Figure 1 about here —

Figure 1 plots the estimates on the year dummy coefficients in the first stage of the AMG estimators. In the present application this represents the average global total factor productivity (TFP) evolution over the sample period. We can detect patterns in this evolution which correspond to global events (e.g. 1970s oil crises) and further obtain a mean estimate for TFP growth of around 1.5% per annum, which also seems reasonable.

### 5 Concluding Remarks

In this note we introduced a very simple panel time series approach, the Augmented Mean Group estimator, which in Monte Carlo simulations was shown to be robust to a great many empirical setups, its performance matching that of the Pesaran (2006) CCE estimators.<sup>5</sup> This aside, we pointed out the dramatic change in simulated performance for 'naïve' estimators (POLS, 2FE) once they were augmented with year dummies — a standard practice in the applied literature — and provided some alternative simulation DGPs where their performance is less favourable and thus more in line with empirical practice using real data. Our empirical illustration highlighted the applicability of the AMG estimator for cross-country production function estimation.

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<sup>&</sup>lt;sup>5</sup>Our focus was on the small sample performance of the AMG estimator and we leave derivation of the asymptotic results for future work.

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## **Tables and Figures**

	(i) <b>BASELINE MODEL</b>			(ii) HETEROGENEOUS TRENDS				
	mean	median	emp. ste <sup>a</sup>	mean ste <sup>b</sup>	mean	median	emp. ste <sup>a</sup>	mean ste <sup>b</sup>
POLS	0.9754	0.9815	0.2139	0.0413	0.9731	0.9688	0.3102	0.0567
2FE	1.0324	1.0312	0.0876	0.0269	1.0277	1.0283	0.1973	0.0552
ССЕР	0.9995	0.9975	0.0333	0.0222	0.9991	1.0003	0.0395	0.0275
FD-OLS	1.0021	1.0015	0.0342	0.0237	1.0025	1.0031	0.0351	0.0250
FE (inf)	1.0000	0.9996	0.0271	0.0161	0.9998	0.9995	0.0280	0.0179
CMG	0.9992	0.9975	0.0338	0.0327	0.9997	0.9997	0.0404	0.0400
AMG(i)	1.0026	1.0008	0.0323	0.0319	1.0049	1.0047	0.0506	0.0322
AMG(ii)	1.0018	1.0004	0.0326	0.0304	1.0090	1.0080	0.0558	0.0432
MG	1.1259	1.1143	0.1825	0.0388	1.1269	1.1185	0.1848	0.0390
MG (inf)	0.9999	0.9989	0.0267	0.0267	1.0017	1.0019	0.0286	0.0281
	(iii) Feedbacks				(iv) $\beta$ -Clubs			
	mean	median	emp. ste <sup>a</sup>	mean ste <sup>b</sup>	mean	median	emp. ste <sup>a</sup>	mean ste <sup>b</sup>
POLS	0.9754	0.9818	0.2138	0.0413	0.5539	0.5537	0.3778	0.1267
2FE	1.0299	1.0285	0.0865	0.0267	1.0224	1.0171	0.1375	0.0388
CCEP	0.9867	0.9851	0.0330	0.0219	1.0017	1.0000	0.0384	0.0257
FD-OLS	0.9149	0.9136	0.0341	0.0235	1.0023	1.0009	0.0401	0.0283
FD-IV	1.0004	0.9993	0.0813	0.0237	1.0011	0.9960	0.0968	0.0284
FE (inf)	0.9924	0.9923	0.0271	0.0159	0.9999	0.9994	0.0357	0.0207
CMG	0.9828	0.9819	0.0333	0.0322	0.9996	0.9991	0.0260	0.0747
AMG(i)	0.9552	0.9541	0.0340	0.0316	1.0045	1.0018	0.0541	0.0750
AMG(i)-IV	0.9959	0.9953	0.0486	0.0317	0.9970	0.9942	0.0512	0.0748
AMG(ii)	0.9511	0.9503	0.0338	0.0303	1.0132	1.0108	0.0625	0.0746
AMG(ii)-IV	1.0015	0.9995	0.0542	0.0303	1.0064	1.0025	0.0603	0.0744
MG	1.1157	1.1043	0.1799	0.0384	1.1260	1.1166	0.1827	0.0788
MG (inf)	0.9888	0.9884	0.0265	0.0265	1.0000	1.0003	0.0174	0.0734

#### Table 1: Simulation Results

Notes: All results are for 1,000 replications in a panel with N = 50 and T = 30. Further results for other combinations of T and N are presented in an Online Appendix. For each estimator we report the mean and median for the 1,000 estimates of average  $\beta$  ( $\overline{\beta}$ ).

<sup>*a*</sup> 'emp. ste' refers to the empirical standard error, the standard deviation of the 1,000 estimates of  $\bar{\beta}$ .

<sup>b</sup> 'mean ste' refers to the sample mean of the estimated standard errors in the 1,000 estimations of  $\bar{\beta}$ , which follow Pesaran and Smith (1995).

**Estimators:** POLS – pooled OLS with period dummies; 2FE - Two-way Fixed Effects (accounts for group- and period-specific effects); CCEP – Pesaran (2006) pooled CCE estimator; FD-OLS – pooled OLS with variables in first difference (includes year dummies); FD-IV – dto. using once-lagged differences to instrument for endogenous*x* $; CMG – Pesaran (2006) Mean Group CCE estimator; AMG(i) – Augmented Mean Group estimator, <math>\hat{\mu}_t^{\bullet}$  imposed with unit coefficient; AMG(ii) – AMG estimator with  $\hat{\mu}_t^{\bullet}$  included as additional regressor; AMG(i)-IV and AMG(ii)-IV – AMG estimators where in the first stage once-lagged differences are used to instrument the endogenous *x*; MG – Pesaran and Smith (1995) Mean Group estimator. Two 'infeasible estimators' (inf) for FE and MG contain the unobserved common factors as additional regressors.

PANEL (A): POOLED MODELS							
estimator dependent variable	[1] POLS ly	[2] <b>2FE</b> ly	[3] CCEP ly	[4] <b>FD</b> Δly			
log capital pw	0.7895 [72.97]**	0.6752 [29.89]**	0.5823 [23.38]**				
$\Delta$ log capital pw				0.3195 [3.61]**			
intercept	1.1474 [8.47]**	2.3786 [9.98]**	0.2489 [0.63]				
Diagnostics							
CRS: F	.96	.00	.00	.72			
FE: $F$		.00	.00				
I(1)	1.00	.78	.00	.00			
CD	.15	.05	.03	.39			
RMSE	.462	.135	.113	.103			

 Table 2: Manufacturing Production Functions (CRS imposed)

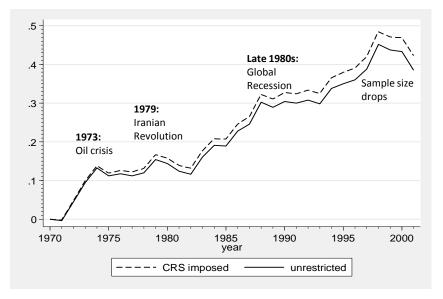
PANEL (B): HETEROGENEOUS PARAMETER MODELS

estimator dependent variable	[1] <b>MG</b> ly	[2] AMG ly- $\hat{\mu}_t^{\bullet}$	[3] AMG ly	[4] CMG ly	[5] <b>CMG</b> ly
log capital pw	0.1789 [2.25]*	0.2896 [3.95]**	0.2982 [3.70]**	0.4663 [6.76]**	0.3125 [3.72]**
common trend			0.8787 [4.39]**		
country trend	0.0174 [5.95]**	0.0001 [0.04]	0.0023 [0.56]		0.0108 [3.09]**
intercept	7.6528 [9.05]**	6.3823 [8.42]**	6.2431 [7.40]**	0.8961 [0.89]	4.7860 [3.66]**
Diagnostics					
I(1)	.00	.00	.00	.00	.00
CD	.00	.96	.30	.02	.82
RMSE	.100	.097	.091	.100	.088

**Notes:** Regressions are for N = 48 countries, n = 1, 194 (n = 1, 128) observations in the levels (first difference) specifications. Values in brackets are absolute *t*-statistics, based on White heteroskedasticity-consistent standard errors in Panel (A) and following Pesaran and Smith (1995) in Panel (B). We indicate statistical significance at the 5% and 1% level by \* and \*\* respectively.

All variables are in logs. Dependent variable: ly — log value-added per worker.  $\Delta$ ly — growth rate of value-added (per worker).  $\hat{\mu}_t^{\bullet}$  in Panel (B) is derived from the year dummy coefficients of a pooled regression (CRS imposed) in first differences (FD-OLS) as described in the main text. Models [1], [2] and [4] in Panel (A) contain T - 1 year dummies (in [4] in first differences). We do not the estimates on the cross-section averages for the CCEP (Panel (A), [3]) and CMG estimators (Panel B, [4] and [5]) to save space.

For all diagnostic tests (except RMSE) we report *p*-values: (i) The null hypothesis for the Wald tests is constant returns. (ii) The *F*-tests in the FE and CCEP regressions maintain the null that fixed effects do not differ across countries. (iii) 'I(1)' reports results for a Pesaran (2007) CIPS test with 2 lags, null of nonstationarity (full results available on request). (iv) The Pesaran (2004) CD test has the null of cross-section independence. Due to data restrictions (unbalanced panel with missing observations) we are forced to drop 2 (8) countries from the sample to compute this test for the levels (FD) residuals. (v) RMSE is the root mean squared error.



### Figure 1: Evolution of average TFP

**Notes:** Derived from results in column [4], Panel (A) of Table 2. Result for unrestricted specification not presented.